Dynamic Friction Parameter Identification Method with LuGre Model for Direct-Drive Rotary Torque Motor

Article in Mathematical Problems in Engineering · March 2016
Impact Factor: 0.76 · DOI: 10.1155/2016/6929457
Research Article

Dynamic Friction Parameter Identification Method with LuGre Model for Direct-Drive Rotary Torque Motor

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Received 2 December 2015; Revised 22 February 2016; Accepted 6 March 2016

Academic Editor: Roque J. Saltarén

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Attainment of high-performance motion/velocity control objectives for the Direct-Drive Rotary (DDR) torque motor should fully consider practical nonlinearities in controller design, such as dynamic friction. The LuGre model has been widely utilized to describe nonlinear friction behavior; however, parameter identification for the LuGre model remains a challenge. A new dynamic friction parameter identification method for LuGre model is proposed in this study. Static parameters are identified through a series of constant velocity experiments, while dynamic parameters are obtained through a presliding process. Novel evolutionary algorithm (NEA) is utilized to increase identification accuracy. Experimental results gathered from the identification experiments conducted in the study for a practical DDR torque motor control system validate the effectiveness of the proposed method.

1. Introduction

The torque motor, especially Direct-Drive Rotary (DDR) torque motor, has been widely utilized in modern industry applications and features rotation blockage, soft mechanical properties, and a wide speed range [1, 2]. Advantages of the motor include high power density, torque/weight ratio, efficiency, rapid response, and small torque ripple [3, 4]. However, designing a high-performance position/velocity tracking controller for the DDR torque motor [5] remains a challenge as multiple factors affecting control precision and dynamic nonlinear friction must be considered [6].

Dynamic friction nonlinearity may be addressed with a properly designed friction compensation controller. Effectiveness of the friction compensation controller is largely dependent on the friction model and accurate friction parameters [7]. The current friction model is divided into two types: the static friction model and the dynamic friction model [8]. The static model reflects the relationship between friction force and the relative movement speed consisting of several distinct parts including static friction, Coulomb friction, viscous friction, and the Striebeck curve effect. Effect of friction, when relative velocity between the two contact surfaces is zero, cannot be described by the static friction model.

The dynamic model features superior practical application value as it reflects the relationship between the friction force and both the speed and displacement, reflecting friction phenomenon more accurately. Primary dynamic friction models are the Dahl model [9], the LuGre model [10, 11], the Leuven model [12, 13], the Generalized Maxwell-Slip (GMS) model [14, 15], and two-state friction model [16]. The Dahl model [9] is derived from the original Coulomb friction model that includes the Striebeck effect and the static friction torque [17]. Canudas de Wit et al. combined the Dahl model with the bristles model to propose a new approach referred to as the LuGre friction model [10, 11, 18]. The LuGre model accurately describes the complex process of static and dynamic properties in friction including presliding displacement, memorial friction, variable static friction, viscous friction, and Striebeck effect. The LuGre model describes the asperities between two contact surfaces by elastic bristles, and true nature of dynamic friction is considered as the result of the average deflection of these bristles. In advance of the LuGre model, the Leuven model [12, 13], and GMS model, authors of [14, 15, 19] modeled the hysteresis behavior of the dynamic friction within presliding regime. Two-state dynamic friction model [16] was also proved to be capable of capturing the hysteresis behavior. However, the modeling of the hysteresis behavior
complicates the friction models significantly and increases the difficulties of implementation in the real-time controls [20]. In addition, the hysteresis behavior is not the essential phenomenon in normal position/velocity control of DDR torque motor. Therefore, the LuGre model is still widely used in motor control systems for dynamic friction compensation.

LuGre model operates based on a group of complex nonlinear functions with six parameters, that is, four static parameters and two dynamic parameters, difficult to identify as coupling exists among the six parameters [21–23]. Difficulty also exists for LuGre parameter identification as the internal state of the model is immeasurable and depends on the previously mentioned unknown friction parameters. Traditional parameter identification methodology is challenged in deriving accurate values of these six parameters in the LuGre model.

A new dynamic friction parameter identification method is proposed in this study for the LuGre model and an identification experiment is conducted for a practical DDR torque motor control system. Static parameters are obtained through a series of constant velocity experiments, while dynamic parameters are obtained by presliding process. Novel evolutionary algorithm [24] (NEA) is utilized to increase identification accuracy and optimization speed by employing Time Variant Mutation (TVM) operator. The proposed method is applied to the practical DDR torque motor control system and the LuGre parameters are obtained by using the proposed identification method. Experimental results validate the effectiveness of the proposed method.

2. Experimental Setup and Mathematical Modeling

2.1. Experimental Setup and Modeling of DDR Torque Motor. The experimental setup of DDR torque motor motion control system for dynamic friction parameter identification is first presented (Figure 1).

The torque motor studied is a current-controlled DDR torque motor D143M by Danaher driven by a commercial digital servo amplifier S620 by Danaher. A Heidenhain high-resolution rotary encoder ECN113 with Heidenhain PC counter card IK220 is installed to measure the motor rotary displacement. Rotary velocity can then be calculated by the derivative of rotary displacement signal. Rotary displacement and velocity signals may be utilized here to identify dynamic friction parameters and online estimate friction torque. Original designed real-time control software based on RTX real-time operating system and LabWindows/CVI is applied to control and monitor the torque motor system with sampling frequency selected as $f_s = 2$ kHz.

Frequency response bandwidth of the motor amplifier is typically higher than 1000 Hz; however, the mechanical dynamics of the DDR torque motor system generally does not exceed 100 Hz. Disregarding the electrical dynamics of the amplifier in normal operating conditions of the DDR torque motor is then reasonable. Input saturation may then also be ignored when the torque motor operates under normal conditions. The relationship of the electromagnetic torque $T_{em}$ and input control voltage $u$ to the motor amplifier, with the above simplifications, may be represented by the following equation [25]:

$$T_{em} = K_m u$$  \hspace{1cm} (1)

where $K_m$ is a proportional coefficient from input control voltage $u$ to electromagnetic torque $T_{em}$.

Considering friction torque and external disturbances, the dynamics of torque motor may be described as

$$T_{em} = K_m u = J_m \dot{\omega}_m + B_m \omega_m + T_d(t) + T_f,$$  \hspace{1cm} (2)

where $J_m$ is total inertia of motor rotator and output shaft; $B_m$ is damping coefficient; $\omega_m$ is rotational velocity of torque motor; $T_d(t)$ represents lumped effect of external disturbances; $T_f$ represents the combination of dynamic friction effects and will be formulated later.

Motor parameter identification experiments are then performed to attain nominal values of system parameters. Identification results are $K_m = 37.7$ Nm/V, $J_m = 0.045$ kg·m², and $B_m = 2.16$ Nm/(rad/s).

2.2. Dynamic Friction Description with LuGre Model. The dynamic friction $T_f$ will be formulated in this subsection by utilizing the LuGre friction model, a nonlinear dynamic friction model, widely utilized in mechanical and servo systems. Elastic bristles are employed by the LuGre model to derive asperities between the two contact surfaces on a microscopic scale with the dynamic effects of friction resulting from the average deflection of these bristles (Figure 2). The LuGre model more accurately describes sliding displacement, memorial friction, variable static friction, viscous friction, and Strieber curve effects synchronously.

The mathematical formulation of LuGre model is as follows [10]:

$$T_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \omega_m,$$  \hspace{1cm} (3)

$$\dot{z} = \omega_m - \frac{[\omega_m]}{g(\omega_m)} \cdot z$$  \hspace{1cm} (4)

$$\sigma_0 g(\omega_m) = f_c + \left( f_s - f_c \right) e^{-\left(\omega_m/v_z\right)^2},$$  \hspace{1cm} (5)

where $z$ is the internal state of LuGre friction model, representing the average deflection of bristles between two...
contact surfaces; \( f_c, f_s, \sigma_2, \) and \( \nu \) are four static parameters, representing Coulomb friction, stiction friction, viscous coefficient, and Stribeck velocity, respectively; \( \sigma_0 \) and \( \sigma_1 \) are two dynamic parameters and \( \sigma_0 \) is the average stiffness coefficient of bristles while \( \sigma_1 \) is the average damping coefficient of bristles, respectively; \( \sigma_0 g(\omega_m) \) describes Stribeck effect.

According to the mathematical expressions of LuGre model (3)–(5), the internal relationship of LuGre model may be as presented in Figure 3.

The average deflection of bristles will obviously reach a relatively stable state (meaning \( \dot{z} = 0 \)) when relative velocity exceeds a certain value. The steady-state bristle average deformation \( z_{ss} \) may be described as

\[
z_{ss} = g(\omega_m) \text{sgn}(\omega_m),
\]

where \( \text{sgn}(\omega_m) \) is the sign function and may be expressed as

\[
\text{sgn}(\omega_m) = \begin{cases} 
1, & \omega_m > 0 \\
0, & \omega_m = 0 \\
-1, & \omega_m < 0 
\end{cases}
\]

Thus, in the steady state, friction torque may be derived by the following formula:

\[
T_f = T_f(z = \omega_m - \frac{\omega_m}{g(\omega_m)} z, \frac{1}{s} z, T_f = \sigma_0 z + \sigma_1 z + \sigma_2 \omega_m)
\]

\[
\sigma_0 g(\omega_m) = f_c + (f_s - f_c) e^{-\omega_m/\nu^2}
\]

The LuGre model, as detailed by the LuGre model expression, includes four static parameters and two dynamic parameters. However, identification of these parameters presents a challenge with increased difficulty for identification of dynamic parameters, an issue solved in the following section.

3. NEA Based Parameter Identification Technique

The NEA based parameter identification technique will be presented in this section to solve the friction parameter identification issue for the LuGre model in the practical DDR torque motor control system.

3.1. Novel Evolutionary Algorithm. Evolutionary algorithm is a widely utilized optimization method [18, 26]. NEA is the development of evolutionary algorithm by utilizing the dynamic TVM operator to improve algorithm speed and precision [24]. TVM operator may produce rapid changes at initial stages of evolution and precise changes at final stages; therefore, it is believed that TVM could improve the efficiency of NEA and guarantee rapid convergence.

The basic flow chart of NEA is presented in Figure 4 and the specific steps are described as follows.

(1) Initial Population. According to empirical statistics, an initial population \( P(0) \) can be selected with \( n \) individuals for target variable set \( \Omega \). These \( n \) individuals are generated by a random function within the desired domain of \( \Omega \). Following evaluation of these \( n \) individuals with an objective function, the initial population \( P(0) \) is divided into \( s \) subpopulations with each subpopulation featuring \( n/s \) individuals. The initial population \( P(0) \) is named as parent population for the next generation.

(2) Individual Evaluation. For target variable set \( \Omega \), the individual which makes objective function minimum is selected as the first individual of the \( i \)th subpopulation and is named as elite individual \( \Theta_{i,mom}^j \) (mother) at \( t \)th generation, expressed as

\[
\Theta_{i,mom}^j = [f_c^j(i,\max), f_s^j(i,\max), \sigma_2^j(i,\max), \nu_s^j(i,\max)].
\]
The average of other individuals then in the $i$th subpopulation excluding $\Theta_{i,\text{mom}}^\prime$ is defined as mean individual $\Theta_{i,\text{dad}}^\prime$ (father), expressed as
\begin{equation}
\Theta_{i,\text{dad}}^\prime = \left[ f_{i,l \text{,mean}}^r, f_{i,l \text{,mean}}^s, \sigma_{i,l \text{,mean}}^r, v_{i,l \text{,mean}}^r \right].
\end{equation}

(3) Crossover Operation. Two offspring $\delta_{i,1}^l$ and $\delta_{i,2}^l$ are generated in this step by crossover operation for each subpopulation as
\begin{equation}
\delta_{i,1}^l = \alpha_1 \Theta_{i,\text{mom}}^\prime + (1 - \alpha_1) \Theta_{i,\text{dad}}^\prime = \left[ \alpha_1 f_{l,i \text{,max}}^r \\
+ (1 - \alpha_1) f_{l,i \text{,mean}}^r, \alpha_2 f_{l,i \text{,max}}^s \\
+ (1 - \alpha_2) f_{l,i \text{,mean}}^s, \alpha_3 \sigma_{l,i \text{,max}}^r \\
+ (1 - \alpha_3) \sigma_{l,i \text{,mean}}^r, \alpha_4 v_{l,i \text{,max}}^r \\
+ (1 - \alpha_4) v_{l,i \text{,mean}}^r \right],
\end{equation}
\begin{equation}
\delta_{i,2}^l = (1 - \alpha_1) \Theta_{i,\text{mom}}^\prime + \alpha_1 \Theta_{i,\text{dad}}^\prime = \left[ (1 - \alpha_1) f_{l,i \text{,max}}^r \\
+ \alpha_1 f_{l,i \text{,mean}}^r, (1 - \alpha_2) f_{l,i \text{,max}}^s \\
+ \alpha_2 f_{l,i \text{,mean}}^s, (1 - \alpha_3) \sigma_{l,i \text{,max}}^r \\
+ \alpha_3 \sigma_{l,i \text{,mean}}^r, (1 - \alpha_4) v_{l,i \text{,max}}^r \\
+ \alpha_4 v_{l,i \text{,mean}}^r \right],
\end{equation}
where $\alpha_k$, $k = 1, 2, 3, 4$, is a random coefficient in $[0, 1]$ and will be recreated for each individual.

For each subpopulation, the elite individual (mother) and the mean individual (father) are chosen and combined. This operation ensures that genes of the elite individual exhibit superior opportunity for the next generation.

(4) Mutation. Mutation is a genetic operator which maintains the genetic diversity and acts as a random variation in a certain range for each subpopulation:
\begin{equation}
\delta_{i,1}^l, \delta_{i,2}^l = \left[ \delta_{i,1}^l, \delta_{i,2}^l, \delta_{i,1}^s, \delta_{i,2}^s \right] = \left[ \delta_{i,1}^l, \beta_1 \lambda(t), \delta_{i,1}^s, \beta_2 \lambda(t) \right],
\end{equation}
where $\delta_{i,1}^l, \delta_{i,2}^l$ are two new offspring generated by the TVM operator and $\beta_k$, $k = 1, 2, 3, 4$, is a random coefficient in $(0, 1)$ and regenerated for each subpopulation. $\lambda(t)$ is a generating function in TVM operation, given as
\begin{equation}
\lambda(t) = \left[ 1 - r^{(1-i/T)^y} \right],
\end{equation}
where $r$ is a random coefficient in $[0, 1]$, $T$ is the maximum number of evolutionary generations, and $y$ is a real-valued parameter to determine the degree of dependency.

(5) Individual Assessment. Each offspring is evaluated by the objective function after mutation operation.

(6) Alternate Generation. The $n^{-1}$th parental generation and the $n$th subgeneration at this stage are integrated together and reordered according to objective function with best $n$ individuals selected as the next generation parent.
Table 1: Static parameters identification results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>Nm</td>
<td>6.975</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Nm</td>
<td>8.558</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Nm/(rad/s)</td>
<td>1.819</td>
</tr>
<tr>
<td>$v_s$</td>
<td>rad/s</td>
<td>6.109 x 10^{-2}</td>
</tr>
</tbody>
</table>

Dynamic parameters $\sigma_0$ and $\sigma_1$ are also optimized utilizing the NEA method. Define a desired set of dynamic parameters as

$$\Omega_d = [\sigma_0, \sigma_1]^T.$$  

Then define the identification error:

$$e_d(\Omega_d, t_j) = \theta_m(t_j) - \theta_d(\Omega_d, t_j),$$  

where $\theta_m(t_j)$ is an angle output of the torque motor at the $t_j$ moment; $\theta_d(\Omega_d, t_j)$ is an angle output of the parameter identification at the $t_j$ moment.

Application of NEA to identify the static parameters allows straightforward estimation of the four initial parameters from the Stribeck curve shape. Estimating the initial value of dynamic parameters $\sigma_0$ and $\sigma_1$, however, from the presliding process curve of torque motor in the identification process of dynamic parameters is difficult. A more reasonable initial estimate for dynamic parameters is then necessary to optimize dynamic parameters by NEA. Realistically, in the presliding process of the torque motor, the input voltage signal is less than the starting torque of torque motor and changes only gradually; thus it is reasonable to disregard the acceleration of motor rotor and the change rate of LuGre model internal state. System dynamics and LuGre model may be transformed into

$$k_m u \approx T_f \approx \sigma_0 z$$  

$$\sigma_2 g(\omega_m) = f_s$$  

$$\dot{z} = \omega_m - \frac{\omega_m}{g(\omega_m)} z.$$  

Suppose this experiment is starting from zero position (i.e., $z(0) = 0$), so (23) can be rewritten as

$$\dot{z} = \frac{d}{dt} = \omega_m - \frac{T_f}{f_s} |\omega_m|,$$  

where rotary velocity $\omega_m$ can be calculated by differentiating the rotary displacement signal of the encoder and friction torque may be calculated by (21). A slowly varying ramp signal $u(t) = k_c t$ is input to the torque motor system until the motor starts. When $\omega_m > 0$, the average deflection of the bristles between the contact surfaces may be obtained by integrating (24) as

$$z(t) = \theta_m(t) - \theta_m(0) + \frac{k_m k_c}{2 f_s} \left( \theta_m(t) + \int_0^t \theta_m(\tau) d\tau \right).$$  

A data group of calculated $z(t)$ can be computed for specified time interval $(0, T)$, and the vectors composed of $z(t)$ are marked as $\mathbf{Z}$ while the vectors composed of $u(t)$ are marked as $\mathbf{U}$. Then the initial value of the dynamic parameters $\sigma_0$, by averaging this data group, may be calculated by

$$\sigma_0^0 = \frac{\mathbf{Z}^T \mathbf{U}}{\mathbf{Z}^T \mathbf{Z}}.$$  

3.3. Identification of Dynamic Parameters in LuGre Model.

The process from a static state to a distinct movement between the two contact surfaces is referred to as the presliding process. LuGre dynamic friction model also explains the presliding process. No obvious movement exists between the two contact surfaces in the presliding process, though lack of obvious movement is not indicative of displacement absence, as even miniscule displacement between the two contact surfaces may be considered as bristles deflection. Friction in the presliding process is mainly composed of two parts, that is, $\sigma_0 z$ and $\sigma_1 \dot{z}$, according to (3). The presliding process in DDR torque motor may be applied then to identify the two dynamic parameters in LuGre model, that is, $\sigma_0$ and $\sigma_1$.

The effect of dynamic parameters $\sigma_0$ and $\sigma_1$ is strengthened in the presliding process in this experiment as the tested torque motor is controlled by the motor driver’s inner current loop only. Input voltage signal $u(t)$ to the motor driver is set to be a slowly varying ramp signal:

$$u(t) = k_c t,$$  

where $k_c > 0$ is a miniscule gradient coefficient and $t$ is time. So under such an input voltage signal, the motor driver will generate a slowly varying electromagnetic torque $T_{em}$ in the motor. The motor is in the presliding motion state before $T_{em}$ exceeds the starting torque of torque motor. Experimental data of input voltage signal and output rotary displacement in this presliding process experiment may be utilized for the identification of two dynamic parameters $\sigma_0$ and $\sigma_1$. Four identified static parameters of LuGre model are also employed for the identification of dynamic parameters.

![Figure 5: Identified Stribeck curve for static parameter identification.](image-url)
The initial value of $\sigma_0$ as $\sigma_0^0 = 2000 \text{ Nm/rad}$ is obtained by applying the above method to the practical torque motor system.

The presliding process of torque motor is selected to estimate the reasonable initial value of other dynamic parameters $\sigma_1$. No obvious movement of motor rotor existed in this case; however, there exists a slight deflection of the bristles between two contact surfaces with the average deflection of the bristles equal to the rotary displacement of motor rotor; thus $\theta_m = z$ and $\omega_m = \dot{z}$, where $\theta_m$ could be measured by the high-resolution rotary encoder and $\omega_m$ could be obtained by the differential of $\theta_m$. When the torque motor is close to zero position, system dynamics and LuGre model may be expressed as

$$m\ddot{\theta}_m + (\sigma_1 + \sigma_2) \dot{\theta}_m + \sigma_0 \theta_m = u(t). \quad (27)$$

Then the transfer function of (27) is

$$\frac{\theta_m(s)}{u(s)} = \frac{1}{ms^2 + (\sigma_1 + \sigma_2)s + \sigma_0}, \quad (28)$$

where $s$ is a Laplace variable. As reported in [21], it is suitable to describe the combination parameter $(\sigma_1 + \sigma_2)$ by applying the concept of damping coefficient. The initial value $\sigma_1^0$ of dynamic parameter $\sigma_1$, as a result, may be obtained from (28) as

$$\sigma_1^0 = 2\xi m \sqrt{\frac{\sigma_0}{m} - \sigma_2}, \quad (29)$$

where $\xi$ is optimal damping ratio.

Applying the above method to the practical torque motor system, the initial value of $\sigma_1$ parameter is obtained as $\sigma_1^0 = 40 \text{ Nm s/rad}$.

The objective function of the dynamic parameter identification is defined as

$$I_d = q_1 \sum_{i=1}^{N} \left| e_d (\Omega_d, t_i) \right|^2 + q_2 \max \left| e_d (\Omega_d, t_i) \right|, \quad (30)$$

where $q_1$ and $q_2$ are two weight coefficients and the identification goal is to minimize $I_d$. Using the initial values of dynamic parameters $(\sigma_0^0, \sigma_1^0)$ and static parameters $\Omega_1$ previously obtained, NEA method is also applied to derive the optimal values of dynamic parameters $\sigma_0$ and $\sigma_1$ by minimizing $I_d$.

Experimental and identification results of the presliding process are revealed in Figure 6. According to these results and NEA optimization, the dynamic parameter identification results are $\sigma_0 = 2750 \text{ Nm/rad}$ and $\sigma_1 = 45.2 \text{ Nm s/rad}$.

### 4. Online Friction Estimation Experiments

In order to further illustrate the proposed parameter identification method, two online friction estimation experiments are carried out, in which the LuGre model with the identified parameters is used. The DDR torque motor is working in the angle control model with a position servo controller.

Two experiments are executed. In the first one, the desired output angle for the DDR torque motor is given by a 1.0 Hz sinusoidal signal with 0.04 rad amplitude. The online friction estimation results are shown in Figure 7, which is calculated by using the LuGre model with the identified parameters.

In the second experiment, the desired output angle for the DDR torque motor is given by a 0.5 Hz sinusoidal signal with 0.15 rad amplitude. The online friction estimation results are shown in Figure 8.

These two friction estimation experiments illustrate that the LuGre model is effective to estimate the dynamic friction in the practical system, and this online estimation could be further utilized for the purpose of friction compensation. Experimental results also validate the effectiveness of the proposed parameter identification method for the LuGre model.

### 5. Conclusion

A new dynamic friction parameter identification method is proposed in this study for LuGre model. NEA technique is utilized to increase identification accuracy and an identification experiment is conducted for a practical DDR torque
motor control system. The four static parameters of LuGre model are identified through a series of constant velocity experiments while two dynamic parameters of LuGre model are obtained through the presliding process. Experimental results validate the effectiveness of the proposed method.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant no. 51305011), the National Basic Research Program of China (973 Program) (Grant no. 2014CB046402), and Program 111 of China.

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