High Performance Adaptive Control of Mechanical Servo System With LuGre Friction Model: Identification and Compensation

LuGre dynamic friction model has been widely used in servo system for friction compensation, but it increases the difficulty of controller design because its parameters are difficult to be identified and its internal state is immeasurable. This paper presents a parameter identification technique based on novel evolutionary algorithm (NEA) for LuGre friction model. In order to settle the practical digital implementation problem of LuGre model, this paper also proposes a modified dual-observer with discontinuous mapping and smooth transfer function. On the basis of the parameter identification results and the modified dual-observer, this paper designs an adaptive control algorithm with dynamic friction compensation for hydraulic servo system. The comparative experiments indicate that the proposed parameter identification technique and the adaptive control algorithm with modified dual-observer are effective with high tracking performance.

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1 Introduction

Development of high performance position tracking control for hydraulic actuator has significant impact on its industrial applications. However, the controller design of hydraulic actuator is a challenging work because hydraulic system is highly nonlinear with parametric uncertainties, unknown modeling errors and external disturbances. In addition, in order to prevent internal or external leakage under high pressure, sealing design will increase friction up to 30% of the nominal actuator torque in some conditions [1]. Friction is a complicated phenomenon which exists in all machines with relative motion; it relies on the physical properties of contact surfaces, relative velocity, and lubrication condition [2]. In practical applications, friction not only seriously affects control precision of servo system, but also leads to undesired stick-slip motion. Many researchers focus on friction compensation aiming to decrease its corresponding influence, whereas it is difficult to do, because there is no effective method to get the exact value of friction [3].

During the past two decades, numerous techniques have been studied to solve difficulties in modeling and compensation of dynamic friction. The classical friction model consists of different components (i.e., static, Coulomb, viscous, and stick-slip friction), while it cannot illustrate the friction effect at zero velocity [4]. In order to solve this problem, Karnopp gave a solution by defining the zero velocity intervals [5]; Haessig and Friedland [6] considered presliding displacement in a bristle model; Dahl model [7] described the spring-like behavior of the contact under static friction, but it did not consider the Strubeck effect and stiction. However, all the researches aforementioned have certain limitations and cannot be effectively applied to friction compensation. Then, Canudas de Wit [8] developed a novel friction model, named as the LuGre model, by integrating the bristle model [6] and Dahl model [7]. LuGre model can describe sliding displacement, memorial friction, variable static friction, and viscous friction synchronously. In LuGre model, there are four static parameters and two dynamic parameters along with an immeasurable internal state describing the average behavior of bristles. Although LuGre model can describe synthetic relation in application, it is difficult to compensate dynamic friction with LuGre model because the friction parameters are difficult to be identified and the internal state is immeasurable. In order to realize friction compensation, there are two issues to deal with: (a) identify the friction parameters; (b) get the internal state z which depends on unknown parameters.

For (a), identification of LuGre model parameters, especially dynamic parameters, is a very challenging task due to immeasurable internal state z and the coupling effect between static and dynamic parameters. Many researchers [9–12] put forward their contributions for the identification of these parameters. This paper presents a parameter identification technique based on novel evolutionary algorithm (NEA) for LuGre friction model, in which the static parameters are identified with constant velocity experiments, and NEA method is utilized to increase identification accuracy based on parameter optimization.

For (b), in order to get the estimate of internal state z, it is necessary to construct observer for dynamic friction compensation. In Ref. [13], a dual-observer structure concept was utilized to solve this nonlinear estimation problem, but it was reported that this kind of observer would become unstable at high speed motions [14]. This paper constructs a modified dual-observer by using discontinuous mapping and smooth transfer function, which can overcome the instability problem of LuGre model observer dynamics.

Hydraulic servo systems are highly nonlinear, in which the flow-pressure nonlinearity, clearance, friction and other unknown modeling errors inevitably exist. No matter how accurate the mathematical model and parameter identification are, it is impossible to capture the exact friction behaviors to realize perfect compensation and tracking control. Therefore, this paper presents a
powerful model-based adaptive control algorithm with dynamic friction compensation, which has features of strong disturbance rejection, performance robustness to model uncertainties, and online learning ability to reduce model uncertainties and to achieve control performance. The experiment results indicate that the proposed control algorithm is effective.

This paper contains the following two main contributions. First, we present a parameter identification technique based on NEA for LuGre friction model, and this identification technique can get quite accurate values of LuGre friction parameters. Second, the instability problem of LuGre model is also settled in this paper by proposing a modified dual-observer for LuGre model.

This paper is organized as follows. Section 2 establishes the dynamic models of hydraulic motor position servo system. Section 3 presents the LuGre parameter identification technique based on NEA. Section 4 proposes the modified dual-observer and the adaptive control law with LuGre friction compensation. Section 5 carries out the comparative experimental certification. Section 6 draws the conclusions.

2 System Dynamic Models

A typical experiment setup of electro-hydraulic servo system is shown in Fig. 1, which consists of a high bandwidth Moog servo valve, a hydraulic motor, a photo-electric rotary encoder, and a differential pressure sensor. The critical issue of this experiment setup is to find out the friction parameters of the hydraulic motor.

The dynamics of above hydraulic motor position servo system can be described as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
m \dot{x}_2 &= u - f - d
\end{align*}
\]

where \( x = [x_1, x_2]^T \) is the state vector consisting of position \( \theta_m \) and velocity \( \theta_m \), \( u \) is control input torque, \( m \) is rotary inertia, \( f \) denotes normalized friction torque, and \( d \) represents constant unknown lumped external disturbances.

For high performance servo system, it is necessary to explicitly consider the effect of nonlinear friction with a suitable dynamic friction model. LuGre model [8] has been widely used in control system, because it can describe major features of dynamic friction, such as presliding displacement, variable breakaway force, viscous friction, and Stribeck effect. LuGre model considers the dynamic effects of friction as a result of the deflection of bristles modeling the asperities between two contact surfaces, in which the internal state \( z \) essentially captures the average behavior of these bristles.

With LuGre model [8], the friction torque function is given as

\[
f(x) = \sigma_0 \dot{z} + \sigma_1 \dot{z} + \sigma_2 x_2 \quad (2)
\]

where \( \sigma_0, \sigma_1 \) represent dynamic friction parameters, viz. stiffness coefficient and damping coefficient respectively. The static parameters \( f_s, z, \sigma_2, \) and \( \nu \) denote Coulomb friction, static friction, viscous coefficient, and Stribeck velocity, respectively. \( z \) is the internal state of friction and \( \sigma_0(x_2) \) describes Stribeck effect.

At steady-state, it is obvious that \( \dot{z} = 0 \). So the steady-state average bristle deflection \( z_{ss} \) can be described as

\[
z_{ss} = g(x_2) \operatorname{sgn}(x_2) \quad (5)
\]

where \( \operatorname{sgn}(x_2) \) is a symbolic function described as

\[
\operatorname{sgn}(x_2) = \begin{cases} 
1, & x_2 > 0 \\
0, & x_2 = 0 \\
-1, & x_2 < 0
\end{cases}
\]

Therefore, the friction torque at steady-state can be given as

\[
f_{ss} = \left[ f_s + (f_s - f_s) e^{-\left(\mathcal{g}/c_0\right)^2} \right] \operatorname{sgn}(x_2) + \sigma_2 x_2 \quad (7)
\]

It is obvious that LuGre model is a dynamic friction model which consists of four static parameters \( f_s, f_s, \sigma_2, \nu \) and two dynamic parameters \( \sigma_0, \sigma_1 \) along with an immeasurable internal state \( z \). Identification of these parameters, especially dynamic parameters, is a very challenging task due to the coupling effect between static and dynamic parameters. Moreover, dynamic friction compensation controller design with LuGre friction model (2) becomes more difficult due to the immeasurable internal state \( z \).

3 Friction Identification

In this section, the parameter identification technique based on NEA for LuGre friction model will be presented. 3.1 Static Parameters Identification. To the electro-hydraulic servo system, the friction−velocity map, which is named as Stribeck curve, is utilized to identify the static parameters of friction under a series of constant velocity motion experiments. On this condition, velocity closed-loop proportional-integral (PI) controller is performed whose objective is to guarantee regular motion without stick-slip [9].

Considering the constant velocity control and neglecting disturbance \( d \), the steady-state friction torque can be described as

\[
f = D_m p_f \quad (8)
\]

where \( D_m \) is volumetric displacement of hydraulic motor and \( p_f \) is load pressure, which can be measured by the differential pressure sensor shown in Fig. 1.

On this condition, the servo system operates at constant velocity \( \{\dot{\theta}_m\}^N_{n=1} \), then the corresponding friction torque can be obtained as \( \{f\}_1^N \) by using Eq. (5). In this paper, we select the number of experiments as \( N = 49 \). All these 49 experiments operate in positive rotational direction firstly, of which 25 experiments are carried out at low velocity region to improve identification accuracy of Stribeck effect. Then the same procedure is performed in negative rotational direction. After that, a relationship of velocity and friction torque can be obtained as well known Stribeck curve.

Define the expected static parameter vector as

\[
\Omega_e = [f_s, f_s, \sigma_2, \nu] \quad (9)
\]
Then define identification error as
\[ e_s(\Omega_s, \hat{\theta}_m) = f(t_i) - f_a(\Omega_s, \hat{\theta}_m, t_i) \]  
(10)
where \( f(t_i) \) is the measured friction torque at moment \( t_i \), and \( f_a(\Omega_s, \hat{\theta}_m, t_i) \) is the modeled friction torque which is determined by Eq. (4) with estimated parameter vector \( \hat{\Omega}_s \).

The objective function for static parameter identification is designed as
\[ J_s = \frac{1}{2} \sum e_s(\beta_2, \hat{\theta}_m)^2 \]  
(11)
Using the NEA, we can get the optimal static parameters of friction which minimizes the objective function \( J_s \).

3.2 Novel Evolutionary Algorithm (NEA). Figure 2 is the flow chart of novel evolutionary algorithm (NEA) [15] for LuGre parameter identification. The steps of static parameters identification has

(1) Initial population: For object variables \( f_c, f_a, \sigma_2, v_s \), we select an initial population \( P(0) \) with \( n = 50 \) individuals according to empirical statistics, and these \( n \) individuals are generated by using a random function within the desired domain of each object variable. After evaluating \( n \) individuals with objective function (8), then the initial population \( P(t) \) is divided into \( s = 5 \) subpopulations, so each subpopulation has \( n/s = 10 \) individuals. This initial population is parent population for next generation.

(2) Individual evaluation: For each object variable, the individual which minimizes objective function (8) is selected to be the first individual of the \( t \) th subpopulation, and it is named as elite individual \( \Theta_{\text{mom}}^t \) (Mom) at generation \( t \) and given as
\[ \Theta_{\text{mom}}^t = [f_{c(max)}^t, f_{a(max)}^t, \sigma_{2(max)}^t, v_{s(max)}^t] \]  
(12)
A mean individual \( \Theta_{\text{dad}}^t \) (Dad) is created from the remaining individuals of the \( t \) th subpopulation excluding \( \Theta_{\text{mom}}^t \) as
\[ \Theta_{\text{dad}}^t = [f_{c(mean)}^t, f_{a(mean)}^t, \sigma_{2(mean)}^t, v_{s(mean)}^t] \]  
(13)
(3) Crossover operation: This step generates two offsprings \( \delta_{i,1}^t, \delta_{i,2}^t \) for each subpopulation by crossover operation as
\[ \delta_{i,1}^t = \beta_i \Theta_{\text{mom}}^t + (1 - \beta_i) \Theta_{\text{dad}}^t \]  
(14)
\[ \delta_{i,2}^t = [1 - \beta_i] \Theta_{\text{mom}}^t + \beta_i \Theta_{\text{dad}}^t \]  
(15)
where \( \beta_k \) represents new offsprings after TVM operation, \( k = 1, 2, 3, 4 \) are random coefficients at \( [0, 1] \) and \( \beta_k \) is sampled anew for each individual.

(4) Mutation: The Mutation phase provides random excursions for offsprings. At this phase, dynamic time variant mutation (TVM) operator can improve tuning efficiency and ensure fast convergence. TVM operation for offspring is defined as
\[ \delta_{i,1}^t, \delta_{i,2}^t = [\delta_{i,1}^t, \delta_{i,2}^t, \delta_{i,3}^t, \delta_{i,4}^t] \]  
(16)
\[ \delta_{i,1}^t = \delta_{i,2}^t + \beta_1 \lambda(t), \delta_{i,3}^t + \beta_2 \lambda(t), \delta_{i,4}^t + \beta_3 \lambda(t) \]  
(17)
where \( \lambda(t) \) is a generating function at TVM step, which is defined as
\[ \lambda(t) = [1 - e^{-(t/T)}] \]  
(18)
where \( T \) is the maximum generation number, \( r \) is a real-valued parameter determining the degree of dependency. The TVM generates rapid changes at initial stages and precise changes at final stages.

(5) Individual evaluation: After mutation operation, each offspring is evaluated by its fitness function.

(6) Alternate generation: In the alternate generation phase, \( n^{-1} \) parents and \( n' \) children are combined and reordered according to their fitness functions. Best \( n \) individuals are selected as parent for next generation.

(7) Check for final condition: This loop continues till final condition is met and the best solution is obtained. Identified static parameters are given in Table 1 and the identification result of Stribeck curve is shown in Fig. 3.

3.3 Dynamic Parameter Identification. In this paper, open-loop experiments are used to identify dynamic parameters, viz. \( \sigma_0 \) and \( \sigma_1 \), because stick-slip motion are highly sensitive to variation of dynamic parameters. The previously obtained static parameters are used to identify dynamic parameters.

![Flow chart of novel evolutionary algorithm](image-url)
The open-loop experiments are carried out with very slow ramp input torque to enhance the effects of dynamic parameters; then, the recorded data are used to search for optimal identification result for dynamic parameters. 

Define the expected dynamic parameter vector as

\[ \Omega_d = [\sigma_0, \sigma_1]^T \]  

(18)

Then the identified error is defined as

\[ e_i(t) = \theta_m(t) - \theta_d(\Omega_d, t_i) \]  

(19)

where \( \theta_m(t_i) \) represents the angle output of physical system at moment \( t_i \) and \( \theta_d(\Omega_d, t_i) \) denotes the output of system model with identified parameters at moment \( t_i \).

In dynamic parameter identification process, the initial value of \( \Omega_0 = [\sigma_0, \sigma_1]^T \) is very important for a precise estimation. We select a preslide process to determine initial value when the system moves from zero position to startup. On this condition, input torque \( u \) is smaller than the breakaway torque and varies very slowly, so acceleration, internal state \( z \) variation and disturbance \( d \) can be neglected. Then, system functions can be transformed to

\[ u \approx f \approx \sigma_0 z \]  

(20)

\[ \sigma_0 g(\dot{\theta}_m) = f_s \]  

(21)

\[ \dot{z} = \dot{\theta}_m - \frac{\dot{\theta}_m}{g(\theta_m)} z \]  

(22)

Assume that the expression starts from zero position (\( z(0) = 0 \)), then, Eq. (22) can be rewritten as

\[ \dot{z} = \frac{\dot{z}}{d} = \frac{\dot{\theta}_m}{f_s} \]  

(23)

In Eq. (23), velocity \( \dot{\theta}_m \) can be measured with encoder and friction torque can be obtained by input signal \( u \) according to Eq. (20). If we input \( u(t) = k_t \dot{t} \) with very small gradient coefficient \( k_t > 0 \) into the system until the motor breaks away, then the average deflection of bristles can be calculated from the integration of Eq. (23) when \( \dot{\theta}_m > 0 \) as

\[ z(t) = \theta_m(t) - \theta_m(0) + \frac{k_t}{2f_s} \left( \theta_m(t) - \int_0^t \theta_m(\tau)d\tau \right) \]  

(24)

With a group of \( z(t) \) calculated in specified time interval \((0, T)\), we can get the initial value of \( \sigma_0 \) by averaging the data as

\[ \sigma_0^0 = \frac{Z^T U}{Z^T Z} \]  

(25)

where \( Z \) and \( U \) are vectors of \( z(t) \) and \( u(t) \), respectively.

In the application of practical hydraulic system, the initial value of \( \sigma_0 \) is \( \sigma_0^0 = 2 \times 10^4 \text{Nm/rad} \).

In order to obtain a reasonable initial value of \( \sigma_1 \), we also consider preslide process. At this case, there is no gross motion, but it exists imperceptible deformation of bristles between contact surfaces, and the average deformation of bristles is equal to the displacement of motor with same direction. So we can consider that \( \dot{\theta}_m \approx z \) and \( \theta_m \approx z \), then Eq. (1) with LuGre model (2) around zero position can be rewritten as

\[ m\ddot{\theta}_m + (\sigma_1 + \sigma_2) \dot{\theta}_m + \sigma_0 \theta_m = u(t) \]  

(26)

Then, we can get the transfer function of Eq. (17) as

\[ \frac{\theta_m(s)}{u(s)} = \frac{1}{ms^2 + (\sigma_1 + \sigma_2)s + \sigma_0} \]  

(27)

where \( s \) represents the Laplace variable. According to literature [10], \( \sigma_1 \) can be described as a damping coefficient, and the initial value \( \sigma_1^0 \) of \( \sigma_1 \) can be gotten as

\[ \sigma_1^0 = 2\zeta m \sqrt{\frac{\sigma_0}{m} - \sigma_2} \]  

(28)

where \( \zeta \) is damping ratio.

In the application of practical hydraulic system, the initial value of \( \sigma_1 \) is \( \sigma_1^0 = 32 \text{Nm/s/rad} \).

Then, the objective function of dynamic parameter identification is defined as

\[ J_d = w_1 \sum_{i=1}^{N} \epsilon_i^2(\Omega_d, t_i) + w_2 \max \{|\epsilon_i(\Omega_d, t_i)|\} \]  

(29)

where \( w_1 \) and \( w_2 \) are two weight coefficients. Optimizing the objective function \( J_d \) with NEA, we can get optimal value of \( \sigma_1, \sigma_2 \), in which initial values \( \sigma_1^0, \sigma_1^0 \) and identification value of \( \Omega_d \) are used.

Figure 4 shows the preslide process of hydraulic motor from zero position to startup, in which the solid line is the experiment result and the dot line expresses the simulation result with identified friction parameters. It is obvious that the identification result is in keeping with real application with identified dynamic parameters \( \sigma_0 = 2.25 \times 10^4 \text{Nm/rad} \) and \( \sigma_1 = 25 \text{Nm/s/rad} \).

4 Controller Design

Although we can get the accurate identification values of LuGre friction parameters, friction torque varies in nonlinear behaviors in real application. So it is necessary to design an appropriate controller to compensate friction variation. A good control algorithm should be suitable to wide parameters variance with strong disturbance rejection and performance robustness. In this section, an adaptive controller with dynamic friction compensation is designed to solve the tracking control problem.

4.1 Dynamic Model Design. Define the tracking errors as

\[ e = x_1 - x_d \]  

(30)
where \( e \) is position tracking error, \( x_d \) is desired motion trajectory, \( r \) is filtered tracking error and \( k_r > 0 \) is a positive gain. Making \( r \) small or converging to zero is equivalent to making \( e \) small or converging to zero, because \( G(s) = e(s)/r(s) = 1/(s + Z) \) is a stable transfer function. So, the rest design work is to make \( r \) as small as possible with a guaranteed transient performance.

Differentiating Eq. (22) and substituting it into the second equation of Eq. (1), the tracking error dynamics can be transformed into

\[
\dot{m} = u - \sigma_0 z - \sigma_1 \left[ x_2 - \frac{|x_2|}{g(x_2)} \right] - \sigma_2 x_2 - d + m \dot{x}_r
\]  

(33)

So, our goal is to minimize \( r \) with friction dynamic (3). It is obvious that internal state \( z \) is immeasurable, we should get the state with an observer as follows:

### 4.2 Modified Dual-Observer for LuGre Model

In order to compensate for the dynamic friction of LuGre model, it is necessary to build an observer to estimate the immeasurable internal state \( z \). With LuGre model, the observer dynamics would be the form of

\[
\dot{z} = x_2 - \frac{|x_2|}{g(x_2)} z + \eta_1 \tau_z
\]  

(34)

where \( \eta_1 \) is observer gain and \( \tau_z \) is observer error correction function to be selected later.

But literature [14] shows that digital implementation of observer, Eq. (25), always leads to control unstable if velocity is larger than a critical value, because the traditional LuGre friction model would become very stiff at high speed. However, we notice that \( \dot{z} \) will converge to zero when the velocity exceeds a critical value, that means \( z \) will be a steady value. So, we can select the maximum average deformation of bristles as steady value of \( z \) to solve the unstable problem aforementioned. Based on discontinuous mapping and smooth transfer function, this paper proposed a modified dual-observer to estimate the immeasurable internal state \( z \) as

\[
\begin{align*}
\dot{z}_0 &= \text{Map} \left[ x_2 - \frac{|x_2|}{g(x_2)} z_0 - \eta_4 r \right] \\
\dot{z}_1 &= \text{Map} \left[ x_2 - \frac{|x_2|}{g(x_2)} z_1 + \eta_1 \frac{|x_2|}{g(x_2)} r \right]
\end{align*}
\]  

(35, 36)

where \( z_0 \) and \( z_1 \) are two estimates of \( z \), and Map(\( \bullet \)) is a discontinuous mapping function which is defined as

\[
\text{Map}(\bullet) = \begin{cases} 
0 & \text{if } \dot{z}_0, \dot{z}_1 = z_{\text{max}} \text{ and } \bullet > 0 \\
0 & \text{if } \dot{z}_0, \dot{z}_1 = z_{\text{min}} \text{ and } \bullet < 0 \\
x(x_2) & \text{otherwise}
\end{cases}
\]  

(37)

where \( z_{\text{max}} = f_i/\sigma_0 \) and \( z_{\text{min}} = -f_i/\sigma_0 \) are observation bounds which correspond to physical bounds of the internal state of dynamic friction, and \( x(x_2) \) is a smooth transfer function designed as

\[
s(x_2) = \begin{cases} 
1 & |x_2| < \nu_1 \\
\frac{1}{2} \cos(\pi |x_2|/\nu_1) + \frac{1}{2} \nu_1 \leq |x_2| < \nu_2 \\
0 & |x_2| > \nu_2
\end{cases}
\]  

(38)

where \( \nu_2 > \nu_1 > 0, \nu_1, \) and \( \nu_2 \) are transfer velocities to be selected based on the particular characteristics of presented system. The character of this modified dual-observer is to stop updating the internal state \( z \) when velocity is high enough and it solves the instability problem of the original observer for LuGre model.

Furthermore, using the similar arguments in Ref. [16], the proposed observer has the following desirable properties [17,18]:

\[
z_{\text{min}} < \dot{z}_0, \dot{z}_1 < z_{\text{max}}
\]  

(39)

\[
z_0 \left\{ \text{Map} \left[ x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_0 - \eta_0 r \right] - x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_0 - \eta_4 r \right\} \leq 0
\]  

(40)

\[
z_1 \left\{ \text{Map} \left[ x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_1 + \eta_1 \frac{|x_2|}{g(x_2)} r \right] - x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_1 + \eta_1 \frac{|x_2|}{g(x_2)} r \right\} \leq 0
\]  

(41)

where \( z_0 = \dot{z}_0 - \dot{z}_0 \) and \( z_1 = \dot{z}_1 - \dot{z}_1 \) are observation errors of Eqs. (26) and (27), respectively.

### 4.3 Adaptive Control Law Design

For the purpose of controller design, the following variables are defined. Let \( \sigma_0, \sigma_1, \sigma_2, d, \) and \( m \) as online estimated value of friction parameters \( \sigma_0, \sigma_1, \sigma_2, \) disturbance \( d, \) and inertia parameter \( m. \) Then, the parameter estimates errors \( \hat{\bullet} \) are defined as \( \hat{\bullet} = \hat{\bullet} - \bullet, \) where \( \bullet \) represents \( \sigma_0, \sigma_1, \sigma_2, d, \) and \( m. \)

With parameter estimates and modified dual-observer (26), (27), the following adaptive controller is proposed as:

\[
u = -k_r r + \sigma_0 \dot{z}_0 + \sigma_1 \left[ x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_1 \right] + \dot{\sigma}_2 x_2 + \dot{d} + m \dot{\dot{x}}_r
\]  

(42)

where \(-k_r r \) is used to stabilize the nominal system and \( k_i \) is a positive design constant. Otherwise, the estimate of friction torque can be given as

\[
\dot{f} = \sigma_0 \dot{z}_0 + \sigma_1 \left[ x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_1 \right] + \ddot{\sigma}_2 x_2
\]  

(43)

Then, substitute the control law (33) into system dynamics (24), the closed-loop error dynamics can be obtained as

\[
\begin{align*}
\dot{m} &= -k_r r + \sigma_0 \dot{z}_0 - \sigma_1 \left[ x_2 - \frac{|x_2|}{g(x_2)} \dot{z}_1 \right] + \sigma_0 \ddot{\dot{z}}_0 + \sigma_2 \ddot{x}_2 + \ddot{\dot{d}} + m \ddot{\dot{x}}_r \\
&+ \ddot{\sigma}_2 x_2 + \ddot{d} + m \ddot{\dot{x}}_r
\end{align*}
\]  

(44)

Consider the closed-loop control system including system (1), LuGre friction model (2)–(4), adaptive controller (33) and modified dual-observer (26) and (27); then, design a Lyapunov function \( V \) as
Make the estimated parameters $\hat{\sigma}_0$, $\hat{\sigma}_1$, $\hat{\sigma}_2$, $\hat{d}$, and $\hat{m}$ to be updated by the following adaptive laws:

$$\dot{\hat{\sigma}}_0 = -\eta_0 \hat{\sigma}_0 \hat{r}$$

$$\dot{\hat{\sigma}}_1 = -\eta_1 \left[x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_1 \right] \hat{r}$$

The time derivative of $V$ can be obtained as

$$\dot{V} = m \hat{r}^2 + \frac{1}{2} c \sigma_0 \hat{z}_0 + \frac{1}{2} \sigma_1 \hat{z}_1 + \frac{1}{2} \sigma_2 \hat{z}_2 + \frac{1}{2} \sigma_0 \hat{z}_0 + \sigma_1 \hat{z}_1 + \sigma_2 \hat{z}_2$$

where $\eta_0$, $\eta_1$, $\eta_2$, $\eta_d$, and $\eta_m$ are positive constants. Then, $\dot{\hat{\sigma}} = \hat{\dot{\sigma}}$, and substituting adaptive laws (39)–(43) into Eq. (38) yields

$$\dot{V} = -k_r \rho^2 + \sigma_0 \hat{z}_0 \rho - \sigma_1 \left| \frac{x_1}{g(x_2)} \right| \hat{z}_1 \hat{r} + \frac{\sigma_2}{\eta_0} \hat{z}_0 + \frac{\sigma_1}{\eta_1} \hat{z}_1 + \frac{\sigma_2}{\eta_2} \hat{z}_2 \hat{r}$$

Considering the relationship of $\hat{z}_0 = \tilde{z}_0 - \hat{z}_0$ and $\hat{z}_1 = \tilde{z}_1 - \hat{z}_1$, and substituting them into Eq. (44), we have

$$\dot{V} = -k_r \rho^2 + \sigma_0 \hat{z}_0 \rho - \sigma_1 \left| \frac{x_1}{g(x_2)} \right| \hat{z}_1 \hat{r} + \frac{\sigma_2}{\eta_0} \hat{z}_0 + \frac{\sigma_1}{\eta_1} \hat{z}_1 + \frac{\sigma_2}{\eta_2} \hat{z}_2 \hat{r}$$

Then, substitute $\hat{z}_0 = \tilde{z}_0 - \hat{z}_0$ and $\hat{z}_1 = \tilde{z}_1 - \hat{z}_1$ above equation, Eq. (45) can be rewritten as

$$\dot{V} = -k_r \rho^2 - \sigma_0 \left| \frac{x_1}{g(x_2)} \right| \tilde{z}_0 \hat{r} - \sigma_1 \left| \frac{x_1}{g(x_2)} \right| \tilde{z}_1 \hat{r} + \frac{\sigma_2}{\eta_0} \tilde{z}_0 - \frac{\sigma_1}{\eta_1} \tilde{z}_1 + \frac{\sigma_2}{\eta_2} \tilde{z}_2 \hat{r}$$

Replacing $\tilde{z}_0$ and $\tilde{z}_1$ with the modified dual observer dynamics (26) and (27), and considering Eq. (31) and Eq. (32), thus the following inequality holds:

$$\dot{V} \leq -k_r \rho^2 - \sigma_0 \left| \frac{x_1}{g(x_2)} \right| \tilde{z}_0^2 - \sigma_1 \left| \frac{x_1}{g(x_2)} \right| \tilde{z}_1^2 \leq -k_r \rho^2$$

for $k_r$, $\sigma_0$, $\sigma_1$, and $g(x_2)$ are positive. Applying the Barbalat's lemma, $r \to 0$ when $t \to \infty$. Since $r$ is related to $e$ via an exponentially stable transfer function, we conclude $e \to 0$ when $t \to \infty$. That means the tracking error will converge to zero and all the signals in the closed-loop are bounded.
5 Application

This paper implements the proposed controller to the electro-hydraulic servo system shown in Fig. 1. The experiment goal is to certify the effectiveness of the proposed control algorithm under parametric uncertainties and dynamic friction effects to achieve desired trajectory tracking in the practical system.

In Fig. 1, the hydraulic motor is controlled by a high bandwidth Moog G761 servo valve. The volumetric displacement $D_m$ of the hydraulic motor is $8.1 \times 10^{-5}$ m$^3$/rad. The supply pressure in these experiments is $1.2 \times 10^5$ Pa. The photo-electric rotary encoder is Heidenhain ROD1080 with PC counter card IK121. The digital system consists of an industrial control computer and a 16-bit PCI-1723 DA card by Advantech. The entire system is controlled by self-developed real-time control software based on RTX real-time operating system and LabWindows/CVI, and control period time is 0.5 ms.

After optimization, the controller gains are given as $k_r = 120$ and $k_i = 50$. The adaptation rates are $\eta_0 = 1 \times 10^4$, $\eta_1 = 100$, $\eta_2 = 10$, $\eta_3 = 500$, and $\eta_w = 0.5$. The initial estimates of LuGre parameters $\sigma_0$, $\sigma_1$, and $\sigma_2$ are chosen as the identification results shown in Sec. 3, while the initial estimates of $d$ and $m$ are 0 Nm and 0.01 Kg m$^2$. The observer gains for dynamic friction compensation are chosen as $u_0 = u_n = 0.01$, initial parameter estimates are chosen as $z_0(0) = z_2(0) = 0$, and the transfer velocities are $v_1 = 0.6$ rad/s and $v_2 = 0.8$ rad/s.

In Eq. (33), the calculated result of this control law is a torque signal while the actual system can just provide control voltage, so it cannot be used directly. Fortunately, the control voltage can be calculated through input torque signal $u$ divided by an experimental coefficient $k_v = 0.478$ V/Nm.

Now, we compare two control algorithms: (C1) adaptive control without friction compensation; (C2) adaptive control with dynamic friction compensation based on LuGre model, in two different classes of trajectory tracking experiments.

5.1 Sinusoidal Motions. First, the desired trajectory is given by a 0.5 Hz sinusoidal signal with 0.2 rad amplitude. Then we get the compared results with tracking errors under C1 and C2 shown in Fig. 5.

As seen from Fig. 5, the tracking performance under C2 is smaller than that of C1 due to effective dynamic friction compensation, especially at peaks and valleys of sinusoidal trajectory. The maximum absolute value of tracking under C1 is 2.639 rad while that of C2 is 0.974 rad, therefore, the proposed controller C2 can reduce the tracking error of 63%.

Since both friction parameters and internal state can be estimated with control algorithm C2, it is easy to obtain friction torque estimation which is calculated by Eq. (43) and is fully utilized in controller C2 as shown in Fig. 6. In order to verify the performance and robustness of the proposed control algorithm in high frequency, the experiment is also carried out under a 10 Hz sinusoidal signal with 0.02 rad amplitude. The tracking errors with the two compared controllers are shown in Fig. 7.

It is obvious that the proposed modified dual-observer is stable at high frequency/high velocity. In Fig. 7, the trend about tracking errors is the same to that in Fig. 5, and 56% of error is reduced by controller C2 according to the maximum absolute value index.

5.2 Point-to-Point Motions. A point-to-point experiment is also executed to examine the ability of the proposed controller. The desired point-to-point trajectory used in this experiment is shown in Fig. 8, and the tracking errors under C1 and C2 are shown in Fig. 9.

In Fig. 9, it is clear that tracking performance of C2 is much better than that of C1, especially when hydraulic motor starts or stops. In this experiment, 59% of error is reduced by controller C2 according to the maximum absolute value index.

The friction torque estimation determined by Eq. (34) with controller C2 in this point-to-point experiment is shown in Fig. 10. Above comparative experiment results indicate that the parameter identification technique based on NEA for LuGre friction parameters is quite accurate, and the modified dual-observer for the internal state $z$ is effective and robust. The well-designed adaptive control algorithm with dynamic friction compensation, by using the identified LuGre parameters and the modified dual-observer, could achieve an excellent trajectory tracking performance.

6 Conclusions

This paper provides the parameter identification technique based on novel evolutionary algorithm for LuGre friction parameters. In order to estimate the immeasurable internal state in LuGre model, this paper investigates a modified dual-observer with discontinuous mapping and smooth transfer function. Considering the variance of friction parameters in real application, an adaptive control algorithm with dynamic friction compensation is developed with rigorous closed-loop stability proofs. The proposed controller is implemented on a hydraulic servo system. The comparative experiment results indicate that the proposed parameter identification technique is effective and the designed adaptive control algorithm with modified dual-observer improves tracking performance.

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References


