Optimization Based on Convergence Velocity and Reliability for Hydraulic Servo System

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Abstract

This article presents an optimal hybrid fuzzy proportion integral derivative (HFPID) controller based on combination of proportion integral derivative (PID) and fuzzy controllers, by which the parameters could be evaluated by global optimization either in convergence velocity or in convergence reliability. Focusing on the nonlinear factors of hydraulic servo system, this article takes advantage of PID and fuzzy logic controller integrated with scaling factors to acquire precise tracking performances. To further improve the performances, it provides new developed optimization with rapid convergence to attain reliable approach probability. Focusing on the performance indicators of evolutionary algorithm, this article presents a new technique to predict reliability of the optimization algorithm. Statistics authenticates the effectiveness and robustness of the optimization. Further, many simulation and experimental results indicate that the optimal HFPID could acquire perfect immunity against parametric uncertainties with external disturbance.

Keywords: adaptive control system; novel evolutionary algorithm; hydraulic control equipment; convergence velocity; convergence reliability; optimization

1. Introduction

The hydraulic servo system (HSS) has found wide range of applications in many precise control industries, thanks to their fast responses, large torque loads, high stiffness, and huge power to mass ratio[1]. However, the HSS is highly nonlinear due to friction, leakage, hysteresis, and nonlinear flows in servo valves[2], which always lead to parametric uncertainties, unknown modeling errors, and external disturbances. Although many nonlinear control techniques were developed to obviate these nonlinear effects over past two decades, they are still in limited use. For example, the adaptive control technique could only deal with small parameter variation, and the sliding mode control was robust against large parameter variation with discontinuous control degradation[3-4]. Many works have suggested that it is necessary to synthesize different control methods to attain a satisfactory control tool for complex nonlinear systems. Hybrid fuzzy proportion integral derivative (HFPID) based on system knowledge is such a kind of integral controller that presents good control performances both in dynamic response and robustness, but it is difficult to have its parameters globally optimized[5-6]. However, as a kind of global optimization method based on biology evolutionism, the genetic algorithm[6-10] has found broad applications in parameter optimization for complex systems.

This article introduces the optimal HFPID with several scaling factors that adopts the novel evolutionary algorithm[11] (NEA) for parameter optimization and attains the convergence reliability with perfect performances and strong robustness against nonlinear influences. Simulation and experimental results indicate that the optimal HFPID could meet the performances required by HSS with high convergence velocity and convergence reliability.

2. Analysis of HSS Nonlinear Factors

A typical rotary HSS consists of a controller, a servo valve, a hydraulic motor, and a sensor, in which the servo valve and the friction are two important nonlinear items[12]. The load flow in a servo valve without
leakage can be described by

\[ Q_f = C_v W x (p_s - p_f) / \rho \]  

(1)

where \( Q_f \) is the load flow, \( C_v \) the discharge coefficient, \( W \) the area gradient, \( x \) the spool displacement, \( p_s \) the supply pressure, \( p_f \) the load pressure, and \( \rho \) the fluid mass density.

It is clear that the nonlinear load flow in a servo valve varies as a square root of the absolute value of the load pressure.

According to the equation of torque balance of HSS, the friction torque \( T_f \) exerts strong influences upon the performances especially at low velocities and in a reversal direction\(^{[2-3]}\). Here, the LuGre dynamic friction model\(^{[13-14]}\) is selected to be an integrated friction model:

\[ T_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 \dot{\theta}_m \]  

(2)

where \( \sigma_0 \) is the stiffness coefficient, \( \sigma_1 \) the damping coefficient, \( \sigma_2 \) the viscous coefficient, \( \dot{\theta}_m \) the angular velocity of motor, \( z \) the dynamics of the friction internal state, whose differential coefficient can be defined as

\[ \frac{dz}{dt} = \dot{z} = \dot{\theta}_m - \frac{[\dot{\theta}_m]}{g(\dot{\theta}_m)} \]  

(3)

where function \( g(\dot{\theta}_m) \) denotes the part of the “steady state” characteristics of the following constant velocity motion model:

\[ g(\dot{\theta}_m) = \frac{1}{\sigma_0} \left[ T_c + (T_c - T_s) e^{-\dot{\theta}_m / \dot{\theta}_s} \right] \]  

(4)

where \( T_c \) is the coulomb friction torque, \( T_s \) the static friction torque, \( \dot{\theta}_s \) the Strubeck velocity.

It is clear that the model of friction torque mainly depends on the nature of the angular velocity. It changes rapidly in the reversal direction as shown in Fig.1.

3. Optimal HFPID Design for HSS

Based on the above-mentioned nonlinear influences, it is difficult for the traditional controller to achieve high immunity against parametric uncertainties in a reversal direction of velocity. The traditional fuzzy controller has good immunity against uncertainties though it is with steady state errors\(^{[5,15]}\). However, PID controller is capable of removing steady state errors effectively. Directed by the compensation limitation of traditional fuzzy controller based on the tracking error \( e \) and its differential \( \dot{e} \), this article presents a HFPID controller which combines scaling factors \( S_c, S_e, S_p \) of fuzzy controller and the proportional coefficient \( K_p \) of proportion integral derivative (PID) in it as shown in Fig.2. With the adjustable parameters \( S_c, S_e, S_p, K_p \), it is easy to compensate the nonlinear influences and get access to perfect performances.

![Fig.2 Schematic of HFPID.](image-url)

In Fig.2, \( u_f \) is the fuzzy control output, \( u_{PID} \) the PID control output, \( u \) the control command, \( K_a \) the gain of hydraulic amplifier, and \( i \) the flow in the servo valve. Here,

\[ u = u_f + u_{PID} \]  

(5)

The outputs in terms of scaling factors can be described by

\[ e_n = S_e e \]  

(6)

\[ \dot{e}_n = S_{\dot{e}} \dot{e} \]  

(7)

where \( e_n, \dot{e}_n \in [-A, A] \), and \( A \in \mathbb{R}^+ \), \( \mathbb{R}^+ \) denotes a set of positive real values.

The output of fuzzy controller

\[ u_f = S_h u_n \]  

(8)

where \( u_n \in [-l_i, l_i] \), \( u_f \in [-H, H] \), and \( l_i, H \in \mathbb{R}^+ \).

Here, the triangular membership functions are used as input and output variables in fuzzy logic controller (see Table 1) to keep high computational efficiency. In Table 1, seven uniform distributed spaces and their fuzzy rules based on system knowledge are derived with the aim to achieve desired responses free of overshoots.

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Fig.1 Velocity friction torque diagram.

Table 1 Fuzzy rule base for HFPID
Next, the most important thing to do is obtain the optimal parameters of HFPID to ensure the perfect performance. To implement the global optimization, the novel evolutionary optimization is adopted to keep fast convergence and coincident approach probability.

4. Novel Evolutionary Algorithm for Optimal HFPID

The kernel issue of evolutionary algorithm is to find the global optimal values with fast convergence that depends on its crossover operator and mutation operator. This article adopts the NEA shown in Fig.3 to obtain the optimal values of $S'_1, S'_2, S'_3,$ and $K'_p$ that meet the performance requirements.

![Typical flow diagram of novel evolutionary algorithm.](image)

The flow of NEA for optimal HFPID can be described as follows:

1. The initial population $P(0)$ is selected to include $\mu = 30$ individuals for each of object variables $S_x, S_y, S_u,$ and $K_p$ according to the empirical statistics. This is randomly initialized by using a uniform random number (URN) within the desired domains of object variables. This initialized population is considered to be the parent population for next generation after evaluating the individuals $\mu$ to their cost function.

2. Based on the natural ecosystem, the subpopulations-based max-mean arithmetical crossover concept is used to reduce the genetic variations and speed up the operation of algorithm. After ordering the individuals to their cost function, the parent population $P(t)$ is divided into $l = 5$ subpopulations in each generation $t$ so that each subpopulation could have $\mu/l = 6$ individuals. The first individual of each object variables of $i$th subpopulation is selected to be an elite individual $\psi^i_{(t,\text{max})}$ (Mom), in generation $t$, because it maximizes a cost function within $i$th subpopulation, and a mean individual $\overline{\theta}^i_{(t,\text{mean})}$ (Dad) is created from the remaining individuals of $i$th subpopulation excluding the $\psi^i_{(t,\text{max})}$ given by

\[
\overline{\theta}^i_{(t,\text{mean})} = \left( \frac{S^i_x + S^i_y + \cdots + S^6_x}{5}, \frac{S^i_x + S^i_y + \cdots + S^6_y}{5}, \frac{S^i_x + S^i_y + \cdots + S^6_u}{5}, \frac{K^i_p + K^i_p + \cdots + K^i_p}{5} \right)
\]

The crossover operation is destined to produce following two offsprings ($\zeta^i_1, \zeta^i_2$):

\[
\begin{align*}
\zeta^i_1 &= (\alpha_1 S^i_{x,\text{max}} + (1 - \alpha_1) S^i_{x,\text{mean}}) \quad \alpha_2 S^i_{y,\text{max}} + (1 - \alpha_2) S^i_{y,\text{mean}} \quad \\
&\quad \alpha_3 S^i_{u,\text{max}} + (1 - \alpha_3) S^i_{u,\text{mean}} \quad \alpha_4 K^i_{p,\text{max}} + (1 - \alpha_4) K^i_{p,\text{mean}} \\
\zeta^i_2 &= ((1 - \alpha_1) S^i_{x,\text{max}} + \alpha_1 S^i_{x,\text{mean}}) \quad \alpha_2 S^i_{y,\text{max}} + (1 - \alpha_2) S^i_{y,\text{mean}} \quad \\
&\quad (1 - \alpha_3) S^i_{u,\text{max}} + \alpha_3 S^i_{u,\text{mean}} \quad (1 - \alpha_4) K^i_{p,\text{max}} + \alpha_4 K^i_{p,\text{mean}}
\end{align*}
\]

where $S_x, S_y, S_u,$ and $K_p$ represent the object variables to be optimized; $(S^i_{x,\text{max}}, S^i_{y,\text{max}}, S^i_{u,\text{max}}, K^i_{p,\text{max}})$ Mom and $(\overline{S}^i_x, \overline{S}^i_y, \overline{S}^i_u, \overline{K}^i_p)$ the corresponding offsprings. Selected fromURN [0, 1], $\alpha$ should be sampled anew for each selected parameter of the individuals.

3. The mutation phase provides random excursions into new location of search space. For this phase, the dynamic time variant mutation (TVM) operator is used to improve fine local tune and ensure the fast convergence. TVM is defined for a child as

\[
\zeta^{ii}_{1} = \left( \overline{S}^i_x + \sigma(1) \cdot N(0,1), \overline{S}^i_y + \sigma(1) \cdot N(0,1), \overline{S}^i_u + \sigma(1) \cdot N(0,1), \overline{K}^i_p + \sigma(1) \cdot N(0,1) \right)
\]

where $\overline{S}^i_x, \overline{S}^i_y, \overline{S}^i_u, \overline{K}^i_p$ represent the new offsprings, $N(\ast, \ast)$ is the Gaussian random value with zero-mean and unity variance, which is sampled anew for each value of the index $i$, and $\sigma(t)$ is the time-varying mutation step generating function in the generation $t$ defined by

\[
\sigma(t) = 1 - r^{(t-T)\gamma}
\]

where $r$ is selected fromURN [0, 1], $T$ the maximal generation number, $\gamma$ the real-valued parameter determining the degree of dependency on the generations. Apparently, TVM generates high values at initial
stages and low values at final stages. This might violate the domain of selected parameters and, if so, the offspring should be left without mutation as shown in Fig.3.

(4) After mutation operation, each offspring is evaluated with its fitness function. The fitness function adopts the integration performance index integral of time and absolute error (ITAE).

\[ J(\text{ITAE}) = \int_{0}^{T_0} |e(t)|dt \]  

(17)

where \( T_0 \) is the total time interval, for which the function is evaluated, \( |e(t)| \) the absolute error. The smaller the ITAE, the better the performance.

(5) In the alternate generation phase, the parent \( P_{t-1} \) and children \( P_t \) are combined and ordered according to their fitness function. The best \( P \) individuals would be selected for the next generation. This operation continues till the final condition and the best solution is found.

5. Convergence Reliability Analysis Based on Statistical Distribution

Ambiguous as an concept is the performance of optimization algorithm depends on the application and implementation[16]. The key indicators of optimization algorithm are its convergence velocity and convergence reliability. The convergence velocity means how fast the algorithm is capable of finding the best solution[9,11,16]. Fig.4 shows the convergence velocity of NEA. From Fig.4, it is clear that the aforementioned NEA can ensure the optimization of parameters \((p^*, \epsilon, e, u, S^*, S^*_c, K_p^*)\) with rapid and stable convergence.

when \( p(*) \) is the probability that a population of the \((1+1)-\)ES reaches the point \( x^t \) in iteration \( t \), \( x^t \) the optimized solution, and \( f_{\text{opt}} \) the global optimum.

Because there is no exact solution available for a complex nonlinear system like HSS to the simple \((1+1)-\)ES, it is very difficult to ensure the convergence reliability of the optimization algorithm. Therefore, a new technique based on the statistics is presented here to warrant the reliability of the NEA. After the algorithm is operated repeatedly \( N \) times, the fluctuation range could be calculated according to the following relationship:

\[
\delta_f = \begin{cases} 
\frac{x_{\text{max}} - f_{\text{opt}}}{f_{\text{opt}}} < 0.005 \times f_{\text{opt}} = \delta_u \\
\frac{f_{\text{opt}} - x_{\text{min}}}{f_{\text{opt}}} < 0.005 \times f_{\text{opt}} = \delta_l 
\end{cases}
\]  

(19)

where \( f_{\text{opt}} \) —the mean value obtained after \( N \) times—is regarded as the approximate global optimum; \( x_{\text{max}}, x_{\text{min}} \) are the maximum and minimum optimal solutions with specific certainty; \( \delta_u \) and \( \delta_l \) the upper and lower fluctuations and \( \delta \) the fluctuation range for specific confidence intervals of \( I = 90\%, 95\%, \) and \( 99\% \). The performance range required by this particular application is \( \delta < \pm 0.5\% f_{\text{opt}} \). Table 2 lists the fitness values after several trials under \( \mu = 30, l = 5, T = 50, N \) (sample size) = 20.

### Table 2 Quantitive comparison of fitness value for confidence intervals

<table>
<thead>
<tr>
<th>Confidence interval/%</th>
<th>Range of fitness value</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Mean value</td>
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<tr>
<td>90</td>
<td>270.406</td>
</tr>
<tr>
<td>95</td>
<td>270.406</td>
</tr>
<tr>
<td>99</td>
<td>270.406</td>
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</table>

When the confidence interval is \( 99\% \), the NEA will provide the results with fluctuation of only \( \pm 0.092 5\% \) with permitted probability.

6. Applications and Discussion

Let the expected tracking error of HSS be less than \( \pm 0.1 \), then the control performance is simulated with the optimal HFPID. Table 3 shows the system parameters.

Fig.5 shows the tracking errors caused by parametric uncertainties, friction, and disturbance from the optimal HFPID.

It is obvious that the optimal HFPID is robust against the nonlinear factors existing in HSS within the desired range for the optimal values \( S^*, S^*_c, S^*_c, K_p^* \). To illustrate the performances of the optimal HFPID, this article compares it with PID, fuzzy controller, and
fuzzy PID (see Fig. 6).

Table 3 Parameters of HSS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tbody>
<tr>
<td>Frequency $\omega$/Hz</td>
<td>100</td>
</tr>
<tr>
<td>Damp coefficient $\xi$</td>
<td>0.4</td>
</tr>
<tr>
<td>Servo valve gain $K_v$(m·A$^{-1}$)</td>
<td>0.001</td>
</tr>
<tr>
<td>Elastic module $E_y$(N·m$^{-2}$)</td>
<td>$6.86 \times 10^6$</td>
</tr>
<tr>
<td>Volumetric displacement $D_m$/rad</td>
<td>$8.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Total volume $V_{ar}$/m$^3$</td>
<td>$6.81 \times 10^{-4}$</td>
</tr>
<tr>
<td>Leakage coefficient $C_{ml}$(N·s·m$^{-1}$)</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Inertia of motor $J_m$(N·m$^2$)</td>
<td>0.0085</td>
</tr>
<tr>
<td>Supply pressure $p_s$(N·m$^{-2}$)</td>
<td>$1.2 \times 10^7$</td>
</tr>
<tr>
<td>Static friction torque $T_s$(N·m)</td>
<td>260</td>
</tr>
<tr>
<td>Coulomb friction torque $T_c$(N·m)</td>
<td>200</td>
</tr>
<tr>
<td>Stiffness coefficient $k_0$(N·m·rad$^{-1}$)</td>
<td>$12 \times 10^3$</td>
</tr>
<tr>
<td>Damping coefficient $\eta_1$(N·m·s·rad$^{-1}$)</td>
<td>300</td>
</tr>
<tr>
<td>Viscous coefficient $\eta_2$(N·m·s·rad$^{-1}$)</td>
<td>60</td>
</tr>
<tr>
<td>Striebeck velocity $\theta_s$/(rad·s$^{-1}$)</td>
<td>0.1</td>
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</table>

It is obvious that PID can maintain the desired performance up to 30% of the given domain, traditional fuzzy control up to 70%, fuzzy PID up to 87%, and the optimal HFPID 100%. Therefore, the optimal HFPID can achieve perfect control immunity against system uncertainties, friction, and disturbance. To verify its adaptability to the actual application, Fig. 7 shows the experimental results from different controllers indicating that the optimal HFPID offers awfully good resistance against parameter variation. It indicates that only the optimal HFPID could meet the requirements by the system.

7. Conclusions

This article provides the optimal HFPID to deal with the nonlinear factors of HSS through introducing the scaling parameters. NEA is used to fulfill the optimization within the desired performance range. To ensure the convergence reliability of NEA, a technique based on statistics is presented to keep permitted fluctuation after a series of trial times. Simulation and experimental results show that the optimal HFPID offers excellent immunity against parametric uncertainties, friction, and external disturbances with high convergence velocity and convergence reliability.

References


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