Output torque tracking control of direct-drive rotary torque motor with dynamic friction compensation

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Abstract

Accurate output torque tracking control problem of Direct-Drive Rotary (DDR) torque motor is concerned in this paper. An adaptive robust torque control algorithm with LuGre model based dynamic friction compensation is proposed to improve the torque tracking accuracy. Adaptive robust control (ARC) backstepping design technique and Lyapunov stability theorem are employed to design the controller to deal with both structured and unstructured uncertainties in a practical control system. In the ARC algorithm, discontinuous projection based parameter adaptive law is utilized to estimate unknown system parameters, and robust control law ensures the robustness of closed-loop control system. In order to further improve the torque tracking control performance, LuGre model based dynamic friction compensation is designed to estimate and compensate friction torque in the practical torque motor system. The mathematically design procedure of the proposed algorithm is developed. Finally, the proposed algorithm is implemented on a practical DDR torque motor control system. Experimental results with three compared control algorithms validate that the proposed algorithm can effectively improve the torque tracking performance and LuGre model based dynamic friction compensation is also necessary for high-accuracy torque tracking control.

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1. Introduction

In model industry, high-accuracy output torque tracking control is often required for advanced manufacturing (such as manufacturing robots and assembly robots) and load test equipments (such as load simulator for aviation actuator and test bed for combustion engine) [1–4].

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Permanent magnet synchronous motor (PMSM), especially Direct-Drive Rotary (DDR) torque motor, is a perfect candidate for such applications, thanks to the advantages as compact size, rapid response, high torque/weight ratio, high efficiency and high power density [5,6]. Furthermore, DDR torque motor can also avoid some mechanical transmission problems, for example backlash. Nevertheless, it is not an easy work to design high-performance torque tracking controller for DDR torque motor, because many issues need to be addressed, such as known or unknown nonlinearities and external disturbances [7].

For known nonlinearities, feedback linearization technique can be employed [8,9]. Furthermore, adaptive approach based controllers have been widely studied for mechanical systems to achieve better control performance [10–13]. However, these aforementioned approaches required precise mathematical models and dealt with parametric uncertainties or structured dynamics only. But in practical mechanical systems, uncertainties or nonlinearities cannot always be modeled by explicit functions, thus the control performance maybe degrade in the presence of unstructured uncertainties or unknown external disturbances. Robust control [14–16] became another choice to handle with unknown nonlinearities and disturbance observers [17–19] were also discussed to estimate and compensate disturbances. Moreover, adaptive robust control (ARC) proposed by Yao and Tomizuka [20,21] could handle both structured and unstructured uncertainties together in one controller. ARC effectively combines techniques of adaptive control (AC) and deterministic robust control (DRC) and improves performance by retaining the advantages of both AC and DRC. Extensive experimental results of ARC have shown its advantages and improved performance [22–25].

On another aspect, in order to achieve further improved tracking control performance, better compensation need to be designed for some specific nonlinearities in DDR torque motor system, for example nonlinear friction. Friction is a complicated phenomenon which exists in all machines with relative motion. Many researchers focused on friction compensation aiming to decrease its corresponding influence, whereas it is difficult to get the exact value of friction [26]. Fortunately, Canudas de Wit [27] developed a novel friction model, named as the LuGre model, by integrating the bristle model [6] and Dahl model [7]. By using four static parameters and two dynamic parameters along with an internal state, LuGre model can describe sliding displacement, memorial friction, variable static friction, and viscous friction synchronously. However, it is still difficult to compensate dynamic friction with LuGre model, because the friction parameters are difficult to be identified and the internal state is immeasurable. Many researchers put forward their contributions to identify LuGre parameters [28–31]. Moreover, the immeasurable internal state increase the design difficulty of LuGre model based friction compensation controller. This issue also attracted the attentions of many researchers. Dual-observer was proposed to online estimate the immeasurable internal state for dynamic friction compensation [32]. However, as reported in [33], when this kind of observer was implemented in a practical digital control system, it would be unstable at high speed motion. Fortunately, this problem was settled in [31] by introducing discontinuous mapping mechanism and smooth transfer function.

In this study, in order to achieve high precision torque tracking control of DDR torque motor, we propose an adaptive robust torque control algorithm with LuGre model based dynamic friction compensation. Discontinuous projection based parameter adaptive law is utilized to estimate unknown system parameters. Robust control law ensures the robustness of closed-loop control system. Using the same observer technique of [31], the modified dual-observer based on discontinuous mapping mechanism and smooth transfer function is employed to estimate the immeasurable internal state of LuGre dynamic friction. A mathematically design procedure of ARC with dynamic friction compensation is developed by combining the adaptive backstepping
technique and Lyapunov theorem. The stability of the closed-loop torque tracking control system is also analyzed. Then the proposed algorithm is tested on a DDR torque motor system and the LuGre friction parameters of the practical system are identified. Comparative experimental results show that our proposed algorithm can achieve more precise torque control. The results also verify that friction compensation is necessary for high-accuracy torque tracking control. Overall, the high-accuracy torque control results obtained from experiments validate the effectiveness of the propose algorithm in practical torque tracking control system.

The reminder of this paper is organized as follows. Section 2 outlines the DDR torque motor mathematical model and the dynamic friction model is also presented in this section. The proposed adaptive robust torque tracking control algorithm is detailed in Section 3. In Section 4, comparative experiments are presented to illustrate the effectiveness of the proposed algorithm. Conclusions are given in Section 5.

2. Modeling and problem formulation

2.1. DDR torque motor modeling

The torque motor considered here is a current-controlled DDR torque motor driven by a commercial motor amplifier. The structure diagram of output torque tracking control system is shown in Fig. 1. In general, the frequency response bandwidth of motor amplifier is higher than 1000 Hz while the mechanical dynamics of DDR torque motor system is usually not exceeding 100 Hz. Thus, it is reasonable to ignore the electrical dynamics of amplifier in normal operating condition. In addition, input nonlinearities, such as dead-zone and saturation, can be neglected too. With these simplifications, the dynamics of torque motor can be described as

\[ K_m u = J_m \omega_m + T_o + T_f(\omega_m) + T_a(t) \]  

where \( u \) is the input voltage signal of motor amplifier, \( K_m \) is the proportional coefficient from \( u \) to electro-magnetic torque, \( J_m \) is the total inertia of motor rotator and output shaft, \( \omega_m \) is the angular velocity of torque motor, \( T_o \) represents output torque which can be measured by torque sensor,
$T_f(\omega_m)$ represents the combination of stiction, Coulomb friction, viscous friction and Stribeck effect, and it will be formulated in the next subsection, $T_d(t)$ represents the lumped effect of external disturbances.

As shown in Fig. 1, the output torque $T_o$ can be measured by torque sensor and described by an elastic model as

$$T_o = K_s \theta_m + d_{sn} \quad (2)$$

where $K_s$ is the total stiffness coefficient of torque sensor and output shaft, $\theta_m$ is the rotary displacement of motor rotator, $d_{sn}$ represents torque sensor measurement noise of torque signal. Then differentiating Eq. (2), the following function can be obtained:

$$\dot{T}_o = K_s \omega_m + d_1 \quad (3)$$

where $d_1 = \dot{d}_{sn}$.

### 2.2 Dynamic friction model

As mentioned in Section 2.1, the friction torque $T_f(\omega_m)$ denotes the lumped effect of different kinds of friction nonlinearities. For high performance control system, it is necessary to be considered with a suitable dynamic friction model. Fortunately, LuGre model [8] is such a friction model that can describe major features of dynamic friction behaviors. As shown in Fig. 2, LuGre model describes the asperities between two contact surfaces as bristles and considers friction as the result of the deflection of these bristles.

Applying LuGre model [8], the friction torque $T_f(\omega_m)$ can be described as

$$T_f(\omega_m) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \omega_m \quad (4)$$

$$\dot{z} = \omega_m - \frac{z|\omega_m|}{g(\omega_m)} \quad (5)$$

$$\sigma_0 g(\omega_m) = T_c + (T_\sigma - T_c)e^{-(\omega_m/\omega_s)^2} \quad (6)$$

where $z$ is the internal state of friction behavior and it captures the average behavior of these bristles. $T_c$, $T_\sigma$, $\sigma_2$ and $\omega_s$ are four static parameters, viz. Coulomb friction, stiction friction, viscous coefficient and Stribeck velocity respectively. $\sigma_0$, $\sigma_1$ represent dynamic friction parameters, viz. the averaged stiffness of the bristles and damping coefficient of the bristles, respectively.

Fig. 2. Bristle deflections between two contact surfaces.
It is obvious that the internal state $z$ is immeasurable, therefore a well-designed observer is necessary to estimate $z$. However, literature [33] pointed out that traditional observer of $z$ always leads to control unstable if velocity is larger than a critical value. Fortunately, this issue was settled in [31] and a modified dual-observer was proposed for $z$, which is given as

$$\dot{\hat{z}_0} = DM \left[ \omega_m - \frac{\dot{\hat{z}}_0 |\omega_m|}{g(\omega_m)} - \eta_{z_0} r \right]$$ (7)

$$\dot{\hat{z}_1} = DM \left[ \omega_m - \frac{\dot{\hat{z}}_1 |\omega_m|}{g(\omega_m)} + \frac{\eta_{z_1} |\omega_m| r}{g(\omega_m)} \right]$$ (8)

where $\dot{\hat{z}}_0$ and $\dot{\hat{z}}_1$ are two online estimates of $z$, $\eta_{z_0}$ and $\eta_{z_1}$ are two positive parameters, $r$ is an adaptation function, and $DM[\bullet]$ is a discontinuous mapping function which is defined as

$$DM[\bullet] = \begin{cases} 0 & \text{if } \dot{\hat{z}}_0, \dot{\hat{z}}_1 = z_{\text{max}} \text{ and } \bullet > 0 \\ 0 & \text{if } \dot{\hat{z}}_0, \dot{\hat{z}}_1 = z_{\text{min}} \text{ and } \bullet < 0 \\ \text{ST}(\omega_m)\bullet & \text{otherwise} \end{cases}$$ (9)

where $z_{\text{max}} = T_s/\sigma_0$ and $z_{\text{min}} = -T_s/\sigma_0$ are observation bounds which correspond to physical bounds of the internal state $z$, and $\text{ST}(\omega_m)$ is a smooth transfer function designed as

$$\text{ST}(\omega_m) = \begin{cases} 1 & \text{if } |\omega_m| < \omega_1 \\ 0 & \text{if } |\omega_m| > \omega_2 \\ 0.5 \cos(\pi \frac{|\omega_m| - \omega_1}{\omega_2 - \omega_1}) + 0.5 & \text{otherwise} \end{cases}$$ (10)

where $\omega_2 > \omega_1 > 0$ are transfer velocities to be selected based on the particular characteristics of presented system. This dual-observer has the following properties [31]:

(P1) $z_{\text{min}} < \dot{\hat{z}}_0, \dot{\hat{z}}_1 < z_{\text{max}}$

(P2) $\dot{\hat{z}}_0 \left\{ \dot{\hat{z}}_0 - \left[ \omega_m - \frac{\dot{\hat{z}}_0 |\omega_m|}{g(\omega_m)} - \eta_{z_0} r \right] \right\} \leq 0$

(P3) $\dot{\hat{z}}_1 \left\{ \dot{\hat{z}}_1 - \left[ \omega_m - \frac{\dot{\hat{z}}_1 |\omega_m|}{g(\omega_m)} + \frac{\eta_{z_1} |\omega_m| r}{g(\omega_m)} \right] \right\} \leq 0$ (11)

where $\dot{\hat{z}}_0 = \dot{\hat{z}} - \dot{z}_0$ and $\dot{\hat{z}}_1 = \dot{\hat{z}} - \dot{z}_1$ are observation errors of Eqs. (7) and (8), respectively.

2.3. Model design and assumptions

By using LuGre friction model, the system can be described in state-space form as follows:

$$\begin{cases} \dot{x}_1 = K_x x_2 + d_1 \\ \dot{x}_2 = \frac{K_m}{J_m} u - \frac{1}{J_m} x_1 - \frac{1}{J_m} (\sigma_0 z_0 + \sigma_1 \dot{z}_1 + \sigma_2 x_2) + d_2 \end{cases}$$ (12)
where \( \mathbf{X} = [x_1, x_2]^T = [T_o, \omega_m]^T \) represents the state vector of output torque and velocity. \( d_2 \) represents the lumped effect of external disturbances and friction approximation error, and it is given as

\[
d_2 = \left[ \frac{1}{J_m} \left( \sigma_0 z_0 + \sigma_1 \dot{z}_1 + \sigma_2 x_2 \right) - \frac{B_m}{J_m} x_2 - \frac{1}{J_m} T_f(x_2) \right] + \frac{1}{J_m} T_d(t)
\]

(13)

Give desired loading torque \( x_d(t) = T_f(t) \), which is assumed to be known and bounded, furthermore its first and second derivatives \( \dot{x}_d, \ddot{x}_d \) are continuous and bounded. The control objective is to design a control input \( u \) such that system output torque \( y = x_1 \) track \( x_d(t) \) as closely as possible in the spite of various model uncertainties.

In general, parametric uncertainties (such as variations of \( K_s, J_m, K_m \) and LuGre model parameters) and nonlinear disturbances (i.e., \( d_1, d_2 \)) always exist in the practical torque motor control system. Thus, it is necessary to reduce parametric uncertainties influence by using online parameter adaptation. For this purpose, define an unknown parameter set \( \mathbf{\Lambda} \) as

\[
\mathbf{\Lambda} = [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8]^T \in \mathbb{R}^8,
\]

in which \( \lambda_1 = K_s, \lambda_2 = -d_1n, \lambda_3 = K_m/J_m, \lambda_4 = 1/J_m, \lambda_5 = \sigma_0/J_m, \lambda_6 = \sigma_1/J_m, \lambda_7 = \sigma_2/J_m \), and \( \lambda_8 = -d_2n \). Here \( d_1n \) and \( d_2n \) can be thought as nominal values of lumped disturbances \( d_1 \) and \( d_2 \) respectively. Then system dynamics (12) can be rewritten as

\[
\begin{align*}
\dot{x}_1 &= \lambda_1 x_2 - \lambda_2 + \tilde{d}_1 \\
\dot{x}_2 &= \lambda_3 u - \lambda_4 x_1 - \lambda_5 z_0 - \lambda_6 \dot{z}_1 - \lambda_7 x_2 - \lambda_8 + \tilde{d}_2
\end{align*}
\]

(14)

where \( \tilde{d}_1 = d_1 - d_1n \) and \( \tilde{d}_2 = d_2 - d_2n \) represent uncompensated disturbances and modeling errors.

The following practical assumption and notations are made:

**Assumption 1.** The extents of parametric uncertainties are known, and the uncertain nonlinearities are bounded by known function, i.e., parametric uncertainties and uncertain nonlinearities are assumed to satisfy

\[
\mathbf{\Lambda} \in \mathbf{\Omega}_\mathbf{\Lambda} \triangleq \left\{ \mathbf{\Lambda}: \mathbf{\Lambda}_\text{min} \leq \mathbf{\Lambda} \leq \mathbf{\Lambda}_\text{max} \right\}
\]

(15)

\[
|\tilde{d}_1| \in \mathbf{\Omega}_{\tilde{d}_1} \triangleq \left\{ \tilde{d}_1: |\tilde{d}_1| \leq \delta_{\tilde{d}_1} \right\}
\]

(16)

\[
|\tilde{d}_2| \in \mathbf{\Omega}_{\tilde{d}_2} \triangleq \left\{ \tilde{d}_2: |\tilde{d}_2| \leq \delta_{\tilde{d}_2} \right\}
\]

(17)

where \( \mathbf{\Lambda}_\text{min} = [\lambda_{1\text{min}}, \lambda_{2\text{min}}, \ldots, \lambda_{8\text{min}}]^T \) and \( \mathbf{\Lambda}_\text{max} = [\lambda_{1\text{max}}, \lambda_{2\text{max}}, \ldots, \lambda_{8\text{max}}]^T \) are known lower and upper bound constant vectors of \( \mathbf{\Lambda} \). \( \delta_{\tilde{d}_1} \) and \( \delta_{\tilde{d}_2} \) are known functions.

**Notation 1:** \( \bullet_i \) for the \( i \)-th component of vector \( \bullet \), \( \bullet_{\text{min}} \) for minimum value of \( \bullet \), and \( \bullet_{\text{max}} \) for maximum value of \( \bullet \).

**Notation 2:** The operation \( \leq \) for two vectors is performed in terms of corresponding elements of the vectors. For example, \( \mathbf{\Lambda}_\text{min} \leq \mathbf{\Lambda} \) means \( \lambda_{j\text{min}} \leq \lambda_j, \forall j \).
3. Adaptive robust torque tracking controller design

3.1. Discontinuous projection-based adaptive law

The discontinuous projection-type parameter adaptation law is used to update parameters in controller. Let \( \hat{\Lambda} \) denote the estimate of \( \Lambda \) and \( \hat{\Lambda} \) denote estimation error (i.e., \( \hat{\Lambda} = \lambda - \Lambda \)). Specifically, viewing Eq. (15), parameter estimate \( \hat{\Lambda} \) is updated through a parameter adaptation law having the form of

\[
\hat{\Lambda} = \text{DPM}_{\hat{\Lambda}}(\hat{\gamma})
\]

(18)

where \( \hat{\gamma} > 0 \) is a diagonal matrix of adaptation rates and \( \hat{\gamma} \) is an adaptation function which can be designed according to system model. The discontinuous projection mapping \( \text{DPM}_{\hat{\Lambda}}(\bullet) \) can be defined as

\[
\text{DPM}_{\hat{\Lambda}}(\bullet) = \begin{cases} 
0 & \text{if } \hat{\Lambda} = \hat{\Lambda}_{\max} \text{ and } \bullet > 0 \\
0 & \text{if } \hat{\Lambda} = \hat{\Lambda}_{\min} \text{ and } \bullet < 0 \\
\bullet & \text{otherwise}
\end{cases}
\]

(19)

It can be shown that for any adaptation function \( \hat{\gamma} \), the projection mapping used in Eq. (19) has the following properties:

(P1) \( \hat{\Lambda} \in \Theta_{\hat{\gamma}} \Leftrightarrow \left\{ \hat{\Lambda} : \hat{\Lambda}_{\min} \leq \hat{\Lambda} \leq \hat{\Lambda}_{\max} \right\} \)

(P2) \( \hat{\Lambda}^T \left[ \hat{\gamma}^{-1} \text{DPM}_{\hat{\Lambda}}(\hat{\gamma}) - \hat{\gamma} \right] \leq 0, \quad \forall \hat{\gamma} \)

(20)

3.2. Adaptive robust controller design

The backstepping design procedure via ARC Lyapunov functions is applied to solve above torque tracking control problem in the following two steps.

Step 1: Design a virtual control law for the first subsystem.

Define the tracking error of the first subsystem of Eq. (14) as

\[
e_1 = T_s - T_r = x_1 - x_d
\]

(21)

where \( x_d = T_r \) is the desired output torque. Treating \( x_2 \) as an input of the first subsystem of Eq. (14), we can design a virtual control law \( u_1 \) for \( x_2 \) to make \( e_1 \) as small as possible with a guaranteed transient performance that means system output \( x_1 \) can track \( x_d \) as closely as possible.
Differentiating Eq. (21) and considering $\dot{x}_1$, the time derivative of $e_1$ can be given as

$$\dot{e}_1 = \lambda_1 x_2 - \lambda_2 + d_1 - \dot{x}_d$$  \hspace{1cm} (22)

Now we can design the control law $u_1$ and it consists of two parts as follows:

$$u_1 = u_{1a} + u_{1s}$$  \hspace{1cm} (23)

where $u_{1a}$ is an adaptive control item and $u_{1s}$ is a robust control item. By using online parameter adaptation (18), the adaptive control item $u_{1a}$ can be designed as

$$u_{1a} = - \frac{1}{\lambda_1} \left[ \hat{\lambda}_2 - \dot{x}_d \right]$$  \hspace{1cm} (24)

Then we define the input difference of first subsystem as

$$e_2 = x_2 - u_1$$  \hspace{1cm} (25)

$e_2$ is also the output error of the second subsystem of Eq. (14). Substituting Eqs. (23) and (24) into Eq. (22) leads to

$$\dot{e}_1 = \lambda_1 (e_2 + u_{1s}) - \dot{\Phi}_1 + d_1$$  \hspace{1cm} (26)

where $\dot{\Phi}_1 = [u_{1a}, -1, 0, 0, 0, 0, 0]^T$. Herein, the parameter adaptation law (18) is discontinuous with discontinuous projection (19), which cannot be used in control law design directly, because backstepping design needs the control function synthesized to be sufficiently smooth at each step in order to obtain its partial derivatives. In order to solve above design difficulty, we design the robust control law as

$$u_{1s} = - \frac{1}{\lambda_{1_{\text{min}}}} k_{1s} e_1 + \alpha_{1s}$$  \hspace{1cm} (27)

where $k_{1s}$ is a positive gain satisfying

$$k_{1s} \geq \left\| \Sigma_{\Phi_1} \dot{\Phi}_1 \right\|^2 + k_1, \quad k_1 > 0$$  \hspace{1cm} (28)

where $\Sigma_{\Phi_1}$ is a positive definite constant diagonal matrix and $k_1$ is a positive design parameter. Furthermore, $\alpha_{1s}$ is a robust control function which is used to reduce the effect of model uncertainties.

Substituting $u_{1s}$ into Eq. (26), we have

$$\dot{e}_1 = \lambda_1 e_2 - \frac{\lambda_1}{\lambda_{1_{\text{min}}}} k_{1s} e_1 + \lambda_1 \alpha_{1s} - \dot{\Phi}_1 + \ddot{d}_1$$  \hspace{1cm} (29)

Define a positive semi-definite (p.s.d.) function $V_1$ as

$$V_1 = \frac{1}{2} e_1^2$$  \hspace{1cm} (30)
Then the time derivative of $V_1$ is
\[ \dot{V}_1 = e_1 \dot{e}_1 \] (31)

Substituting $\dot{e}_1$ into $\dot{V}_1$ leads to
\[ \dot{V}_1 = e_1 \dot{e}_2 - \frac{\lambda_1}{\lambda_1 \min} k_i \alpha \dot{e}_1^2 + e_1 \left( \lambda_1 \alpha_{1s} - \overrightarrow{A} \cdot \overrightarrow{\Phi}_1 + \tilde{d}_1 \right) \] (32)

Now, the robust control function $\alpha_{1s}$ can be chosen to satisfy the following robust performance conditions:

condition (i) $e_1 \left( \lambda_1 \alpha_{1s} - \overrightarrow{A} \cdot \overrightarrow{\Phi}_1 + \tilde{d}_1 \right) \leq \varepsilon_1$

condition (ii) $e_1 \alpha_{1s} \leq 0$ (33)

where $\varepsilon_1$ is a positive design parameter which can be arbitrarily small, and it represents attenuation level of model uncertainties. Essentially, condition (i) of Eq. (33) represents the fact that $\alpha_{1s}$ is synthesized to dominate the model uncertainties coming from both parametric uncertainties $\overrightarrow{A}$ and uncertain nonlinearities $\tilde{d}_1$ to achieve guaranteed attenuation level $\varepsilon_1$, and condition (ii) is imposed to make sure that $\alpha_{1s}$ is dissipating in nature so that it does not interfere with the functionality of adaptive control law $u_{1a}$.

**Remark 1.** One smooth example of robust control function $\alpha_{1s}$ satisfying conditions (33) can be designed as follows. Let $h_1$ be any smooth function satisfying $h_m \geq \|\overrightarrow{\Phi}_1\| \cdot \|\overrightarrow{A}_m\| + \delta_{d_1}$ where $\overrightarrow{A}_m = \overrightarrow{A}_{\text{max}} - \overrightarrow{A}_{\text{min}}$. Then $\alpha_{1s}$ can be designed as
\[ \alpha_{1s} = -\frac{1}{4\lambda_1 \min \varepsilon_1} h_1^2 e_1 \] (34)

It can be shown that Eq. (34) is satisfied. Other examples of $\alpha_{1s}$ can be found in [10,11,18].

**Step 2:** Design an actual control law for the second subsystem.

Noting Eq. (14), the backstepping design in Step 2 is to synthesize an actual control law $u$ such that $x_2$ tracks desired virtual input $u_1$ given by Eqs. (23), (24) and (27) so as to achieve guaranteed transient performance. This process can be completed by using the same ARC design technique as in Step 1.

Noting Eqs. (14) and (25), the time derivative of $e$ can be given as
\[ \dot{e}_2 = \lambda_3 u - \lambda_4 x_1 - \lambda_5 \dot{z}_0 - \lambda_6 \dot{z}_1 - \lambda_7 x_2 - \lambda_8 + d_2 - u_1 \] (35)

With same techniques in Eq. (23), the actual control input $u$ can be designed as
\[ u = u_2 = u_{2a} + u_{2s} \] (36)
where $u_{2a}$ and $u_{2s}$ are adaptive control law and robust control law, respectively.

Substituting Eqs. (36) and (37) into Eq. (35) leads to

$$
e_2 = \dot{\lambda}_3 u_{2s} - \lambda_1 e_1 - \overrightarrow{\Lambda} \Phi_2 + \lambda_5 \hat{z}_0 + \lambda_6 \hat{z}_1 + \ddot{d}_2$$

where $\Phi_2 = [e_1, 0, u_{2a}, -x_1, -\hat{z}_0, -\hat{z}_1, -x_2, -1]^T$. Then the robust control law can be designed as

$$u_{2s} = -\frac{1}{\lambda_3 \min} k_{2s} e_2 + \alpha_{2s}$$

where $k_{2s}$ is a positive gain to stabilize the nominal system and it satisfies that

$$k_{2s} \geq \left( \frac{1}{\dot{\lambda}_3} \right)^2 + \left( \frac{1}{\dot{\Lambda}} \right)^2 + \left( \frac{1}{\Phi_2} \right)^2 + k_2, \quad k_2 > 0$$

Define a p.s.d. Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} e_2^2$$

And its time derivative can be given as

$$\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2$$

Then substitute $\dot{V}_1$ and $\dot{e}_2$ into $\dot{V}_2$, Eq. (43) can be rewritten as

$$\dot{V}_2 = -\frac{\lambda_1}{\lambda_1 \min} k_{1s} e_1^2 + e_1 (\lambda_1 \alpha_{1s} - \overrightarrow{\Lambda} \Phi_1 + \ddot{d}_1)$$

$$\dot{V}_2 = -\frac{\lambda_3}{\lambda_3 \min} k_{2s} e_2^2 + e_2 (\lambda_3 \alpha_{2s} - \overrightarrow{\Phi_2} + \lambda_5 \hat{z}_0 + \lambda_6 \hat{z}_1 + \ddot{d}_2)$$

Similar to Eq. (33), the robust control function $\alpha_{2s}$ is chosen to satisfy the following robust performance conditions:

condition (i) $e_2 (\lambda_3 \alpha_{2s} - \overrightarrow{\Phi_2} + \lambda_5 \hat{z}_0 + \lambda_6 \hat{z}_1 + \ddot{d}_2) \leq e_2$

condition (ii) $e_2 \alpha_{2s} \leq 0$
where $e_2$ is a design parameter which can be arbitrarily small. As in Remark 1, one example of $\alpha_{2s}$ satisfying Eq. (45) is given as

$$\alpha_{2s} = -\frac{1}{4\lambda_{3\min}e_2}h_2^2e_2$$

(46)

where $h_2$ be any smooth function satisfying $h_2 \geq \|\mathbf{\Phi}_2\|\|\Lambda_m\| + \delta_{f2}$.

3.3. Main results

Let the estimated parameters be updated by adaptive law (18) in which $\gamma$ is chosen as

$$\dot{\gamma} = \dot{\mathbf{\Phi}}_1e_1 + \dot{\mathbf{\Phi}}_2e_2$$

(47)

If controller parameters

$$\Theta_{\Phi_1} = \text{diag}\left\{\theta_{\Phi_{11}}, \theta_{\Phi_{12}}, \theta_{\Phi_{13}}, \theta_{\Phi_{14}}, \theta_{\Phi_{15}}, \theta_{\Phi_{16}}, \theta_{\Phi_{1}}, \theta_{\Phi_{1k}}\right\}$$

$$\Theta_{\Phi_2} = \text{diag}\left\{\theta_{\Phi_{21}}, \theta_{\Phi_{22}}, \theta_{\Phi_{23}}, \theta_{\Phi_{24}}, \theta_{\Phi_{25}}, \theta_{\Phi_{26}}, \theta_{\Phi_{2}}, \theta_{\Phi_{k}}\right\}$$

$$\Theta_{\Lambda_1} = \text{diag}\left\{\theta_{\Lambda_{11}}, \theta_{\Lambda_{12}}, \theta_{\Lambda_{13}}, \theta_{\Lambda_{14}}, \theta_{\Lambda_{15}}, \theta_{\Lambda_{16}}, \theta_{\Lambda_{1}}, \theta_{\Lambda_{1k}}\right\}$$

are chosen such that

$$\theta_{\Phi_{1j}}\theta_{\Lambda_{2j}} \geq \sqrt{1/2}$$

(48)

$$\theta_{\Phi_{2j}}\theta_{\Lambda_{2j}} \geq \sqrt{1/2}$$

(49)

where $\theta_{\Phi_{1j}}$ and $\theta_{\Phi_{2j}}$ are the $j$th diagonal elements of $\Theta_{\Phi_1}, \Theta_{\Phi_2}$ and $\Theta_{\Lambda_1}$, respectively. Then the adaptive robust torque control law (23) and (36) with the parameter adaptive law (18) guarantees the following:

(A) In general, control input is bounded and the closed-loop system is globally stable with $V = V_2$ bounded by

$$V(t) \leq \exp(-k_Vt)V(0) + \frac{e_V}{k_V}\left[1 - \exp(-k_Vt)\right]$$

(50)

where $k_V = 2\min\{k_{1s}, k_{2s}\}$ and $e_V = \epsilon_1 + \epsilon_2$.

(B) If after a finite time $t_0$, $\tilde{d}_1 = 0$ and $\tilde{d}_2 = 0$, i.e., in the presence of parametric uncertainties only, then, in addition to results in (A), asymptotic output tracking (or zero final tracking error) is also achieved.

4. Experimental results

In order to illustrate the efficiency of the proposed algorithm, a practical torque tracking control experiment environment has been setup. The proposed control algorithm was
implemented in the practical digital control system and comparative experiment results will be shown as follows.

4.1. Experiment setup and parameter identification

The experimental setup of DDR torque motor control system is showed in Fig. 3. The tested motor is a Direct-Drive Rotary torque motor D143M by Danaher and it is driven by a Danaher digital servo amplifier S620. A high-precision torque sensor AKC-17 is installed to measure the output torque of the tested motor. A Heidenhain high-resolution rotary encoder ECN113 with Heidenhain PC counter card IK220 are used to measure the rotary displacement of motor and the velocity signal is obtained by the difference of rotary displacement. Here, rotary displacement and velocity are used to online estimate friction torque for dynamic friction compensation. A 16-bit AD/DA multi-function card PCI-1716 by Advantech is used to sample torque signal and to send out control voltage to the motor amplifier. Original designed real-time control software based on RTX real-time operating system and Labwindows/CVI is applied to control and monitor the torque motor system and its sampling frequency is selected as $f_s = 2$ kHz.

Parameter identification experiments of the tested DDR torque motor are performed to get the nominal values of system parameters and the identification results are shown in Table 1.

LuGre model parameters in the tested torque motor are also identified by using the same technique as in [31], and the identification results are shown in Table 2.
According to the above identified parameters and control performance of practical experiments, the lower and upper bounds of parameter estimations are chosen as 
\[
\vec{\Lambda}_{\text{max}} = [2500, 20, 1000, 25, 65000, 1200, 50, 10]^T,
\]
\[
\vec{\Lambda}_{\text{min}} = [1500, -20, 600, 15, 55000, 800, 40, -10]^T.
\]

Then the initial values of parameter estimations are chosen as 
\[
\hat{\vec{\Lambda}}_0 = [1965, 0, 840, 22, 60000, 1000, 45, 0]^T,
\]
and the adaptive rates for parameter estimations are chosen as 
\[
\vec{\Gamma} = \text{diag} \{1, 10, 10, 1.2, 120000, 8000, 50, 100\}.
\]

The parameters of the proposed ARC torque controller are chosen as \(k_{1s} = 1500\), \(k_{2s} = 2.5\), \(\varepsilon_1 = 250\), \(\varepsilon_2 = 150\). Parameters in the dual-observer for LuGre friction model are chosen as \(\eta_{c0} = 0.4\), \(\eta_{ci} = 0.006\). The initial values of internal state estimation are \(\hat{\xi}_0(0) = \hat{\xi}_1(0) = 0\), and transfer velocities are chosen as \(\omega_1 = 0.8 \text{ rad/s}\), \(\omega_2 = 1.0 \text{ rad/s}\).

In order to illustrate the control performance of the proposed ARC torque control algorithm with dynamics friction compensation, the following three algorithms will be compared in two torque tracking control experiments:

**C1**: PID controller without friction compensation and its parameters are chosen as outer torque tracking loop \(k_p=0.7\), \(k_i=0.1\) and \(k_d=0.0001\); inner velocity tracking loop \(k_p=2.5\), \(k_i=0.2\) and \(k_d=0.001\);

**C2**: ARC algorithm without friction compensation;

**C3**: ARC algorithm with dynamic friction compensation.

In these three algorithms, C2 and C3 use the same ARC algorithms that means they use the same control laws and the same control parameters. The difference between C2 and C3 is that C3 use LuGre model based dynamic friction compensation but C2 not.

### 4.2. Low output torque tracking control experiment

In this experiment, the desired loading torque is given by a 1.0 Hz sinusoidal signal with 80 N m amplitude. Then torque tracking errors with three controllers are shown in Fig. 4.

**Table 2**
Identified parameters of LuGre model in torque motor system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_c)</td>
<td>6.975</td>
<td>N m</td>
</tr>
<tr>
<td>(T_s)</td>
<td>8.558</td>
<td>N m</td>
</tr>
<tr>
<td>(\sigma_0)</td>
<td>2750</td>
<td>N m/rad</td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>45.2</td>
<td>N m/(rad/s)</td>
</tr>
<tr>
<td>(\sigma_2)</td>
<td>1.819</td>
<td>N m/(rad/s)</td>
</tr>
<tr>
<td>(\omega_s)</td>
<td>0.06109</td>
<td>rad/s</td>
</tr>
</tbody>
</table>
As seen from Fig. 4, C3 gets the best output torque tracking control performance among the three compared algorithms. Specifically, C2 and C3 have much better tracking performance than C1 thanks to the application of ARC algorithm. This result strongly demonstrates the advantages of ARC algorithm to deal with both structured and unstructured uncertainties in a practical control system. Moreover, C3 achieves further improved tracking control performance than C2, because C3 algorithm employs dynamic friction compensation by using LuGre friction model, then the resulting accurate torque tracking is achieved.

Fig. 5 gives out the online estimations of LuGre model internal state with the modified dual-observer (7) and (8). Then the friction torque can be online calculated by using LuGre model and the estimation result is shown in Fig. 6. The control signal of (C3) is also shown Fig. 7 and it is formed as $u = u_2 = u_{2d} + u_{2e}$. 

Fig. 4. Tracking errors in low output torque tracking control experiment.

Fig. 5. Online estimations of friction internal state in low output torque tracking control experiment.
4.3. High output torque tracking control experiment

In order to further verify the performance of the proposed algorithm, experiment is also conducted to tracking high desired output torque signal which is given by a 0.5 Hz sinusoidal signal with 300 N m amplitude, then the tracking errors with the above three compared controllers are shown in Fig. 8.

By comparing the tracking errors shown in Fig. 8, the same result can be got as in the first experiment. C3 also achieves the best tracking control performance among the three compared controllers.
Fig. 8. Tracking errors in high output torque tracking control experiment.

Fig. 9. Online estimations of friction internal state in high output torque tracking control experiment.

Fig. 10. Online estimation of dynamic friction in high output torque tracking control experiment.
algorithms, thanks to the advantages of both ARC algorithm and LuGre model based dynamic friction compensation.

Fig. 9 gives out the online estimations of LuGre model internal state, then the resulted estimation of friction torque is shown in Fig. 10. The control signal of (C3) in this experiment is shown in Fig. 11.

To sum up, the above two comparative experiment results confirm that the proposed algorithm C3 can achieve excellent torque tracking control performance in practical DDR torque motor control system.

5. Conclusions

An adaptive robust torque control algorithm with LuGre model based dynamic friction compensation has been proposed in this study to achieve high precision torque tracking control for DDR torque motor. ARC backstepping technique and Lyapunov theorem are employed to design the controller for the practical control system. Discontinuous projection based parameter adaptive law can estimate unknown system parameters to deal with structured uncertainties. Robust control law ensures the robustness of closed-loop control system in the case of unstructured uncertainties. LuGre model based dynamic friction compensation method can effectively estimate and compensate friction torque. A mathematically design procedure of the proposed algorithm is developed and the stability of the closed-loop torque tracking control system is analyzed. The proposed algorithm is implemented on the practical DDR torque motor control system and comparative experimental results are carried out. The results validate that the
proposed algorithm can improve the torque tracking performance and LuGre model based
dynamic friction compensation is also effective for high-accuracy torque tracking control.

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